SET (प्रारूप)–2

SECTION (खण्ड)–I

Objective Questions (वस्तुनिष्ट प्रश्न)

Time : [1 Hrs + 15 Min. (Extra)] **Full Marks : 50**

समय : 1 घंटा + 15 मिनट (अतिरिक्त) **पूर्णांक : 50**

From Question no. 1 to 50 these is only one correct answer for each question. You have to mark the correct option from the given options.

[प्रश्न संख्या 1 से 50 तक प्रत्येक प्रश्न के केवल एक उत्तर सही है। प्रत्येक प्रश्न के सही उत्तर को चुनकर उत्तर तालिका में चिह्नित करें] **[50 × 1]**

1. If A and B be matrices of order m × n and n × m respectively, then the order of 2A + 5B will be which of the following ?

यदि A और B क्रमशः m × n और n × m कोटि के आव्युह है, तो निम्न में से कौन-सा $2A + 5B$ का कोटि है ?

(a) m × m (b) m × n (c) n × m (d) Not defined

2. A relation R is defined from A to B by R = {(x, y), where $x \in N$, $y \in N$ and $x + y = 4$ }, Then R is a relation of which type ?

A से B पर एक सम्बन्ध R इस प्रकार परिभाषित है $R = \{(x, y)$, जहाँ $x \in N$, $y \in N$ और $x + y = 4$ }, तो R किस प्रकार का संबंध है ?

(a) Symmetric (सममित) (b) Reflexive (स्वतुल्य)

(c) Transitive (संक्रमक) (d) Both (a) & (b) ((a) और (b) दोनों)

3. Let A be a non-singular matrix of order 2 × 2 then |Adj A| =

माना कि A एक व्युत्क्रमणीय आव्युह है जिसका क्रम 2 × 2 है, तो |Adj A| =

(a) 2|A| (b) |A| (c) $|A|^2$ (d) $|A|^3$

4. Let $A = \{a, b\}$ and $B = \{p, q, r\}$ be two set. Then Total no. of bijective functions from A to B is equal to

माना कि $A = \{a, b\}$ और $B = \{p, q, r\}$ दो समुच्चय है, तो A से B पर बायजेक्टिव फलन की कुल संख्या है ।

(a) 0 (b) 6 (c) 3 (d) (None of these) इनमें से कोई नहीं

5. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ **is equal to (बराबर है ।)**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{9}$ (d) $\frac{\pi}{12}$

6. यदि $x = at^2, y = 2at$ तो $\frac{dy}{dx}$ =

If $x = at^2, y = 2at$ then $\frac{dy}{dx}$ =

(a) t (b) $\frac{1}{t}$ (c) at (d) $\frac{a}{t}$

7. **The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is**

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ का मुख्य मान है ।

(a) $\frac{-2\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{\pi}{3}$

8. **The differential equation of family of lines passing through the origin is**
मूल बिन्दु से गुजरने वाली रेखाओं के परिवार का अवकल समीकरण होगा ।

(a) $x\frac{dy}{dx}=y$ (b) $y\frac{dy}{dx}=x$ (c) $\frac{dy}{dx}=y$ (d) $\frac{dy}{dx}=x$

9. $\int \log e^x \, dx =$

(a) $x\log x + x + k$ (b) $x\log x - x + k$ (c) $\log x + x + k$ (d) $\log x - x + k$

10. **$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ is equal to which of the following.**

$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ निम्न में से किसके बराबर है ।

(a) $e^{\sqrt{x}}$ (b) $\frac{e^{\sqrt{x}}}{2}$ (c) $2.e^{\sqrt{x}}$ (d) $\sqrt{x}.e^{\sqrt{x}}$

11. **The position vector of the middle point of the line joining the points $3\hat{i}+\hat{j}-\hat{k}$ and $\hat{i}+3\hat{j}+\hat{k}$ is**
बिन्दु $3\hat{i}+\hat{j}-\hat{k}$ और $\hat{i}+3\hat{j}+\hat{k}$ को जोड़ने वाली रेखा के मध्य बिन्दु का स्थिति सदिश है ?

(a) $2\hat{i}+2\hat{j}$ (b) $4\hat{i}+4\hat{j}-2\hat{k}$ (c) $2\hat{i}-2\hat{j}-2\hat{k}$ (d) (None of these) इनमें से कोई नहीं

12. **If $\vec{a}$ and $\vec{b}$ are unit vector such that $\vec{a}+\vec{b}$** is also a unit vector. Then the angle between the vector $\vec{a}$ **&** $\vec{b}$ **is**

यदि $\vec{a}$ और $\vec{b}$ इकाई सदिश इस प्रकार है कि $\vec{a}+\vec{b}$ भी एक इकाई सदिश है, तो सदिश $\vec{a}$ और $\vec{b}$ के बीच का कोण है ?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{4\pi}{3}$ (d) $\frac{2\pi}{3}$

13. **The value of $\int_{\partial}^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}} dx$ is equal to which of the following**

$\int_{0}^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}} dx$ का मान निम्न में से किसके बराबर है ?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{\pi}{3}$

14. The maximum value of $\sin x + \cos x$, $0<x<\frac{\pi}{2}$ is

$\sin x + \cos x$ का अधिकतम मान जब $0<x<\frac{\pi}{2}$ है ?

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $\sqrt{\frac{3}{2}}$

15. In which of the following interval the function $f(x) = x^2 - x$ is increasing.
निम्नलिखित में से किस अन्तराल में फलन $f(x) = x^2 - x$ वर्धमान है ?

(a) $\left(\frac{1}{2},\infty\right)$ (b) $\left(-\infty,\frac{1}{2}\right)$ (c) $(-\infty, \infty)$ (d) (None of these) इनमें से कोई नहीं

16. Which of the following is the equation of xy-plane.
निम्नलिखित में xy– तल का समीकरण है ?

(a) $x = 0$ (b) $y = 0$ (c) $x = k$ (d) $z = 0$

17. Which of the following is a vector quantity.
निम्नलिखित में कौन एक सदिश राशि है ?

(a) $\vec{a}\times(\vec{b}.\vec{c})$ (b) $\vec{a}\,(\vec{b}\times\vec{c})$ (c) $\vec{a}.(\vec{b}.\vec{c})$ (d) (None of these) इनमें से कोई नहीं

18. The direction cosine of any line parallel to x- axis is
x– अक्ष के समानान्तर किसी रेखा का दिक् कोज्या है ?)

(a) (1, 0, 0) (b) (1, 1, 1,) (c) (0, 1, 0) (d) (None of these) इनमें से कोई नहीं

19. The probability distribution of a random variable x is given by :
एक यादृच्छ चर x का प्रायिकता वितरण इस प्रकार है ?

x	1	2	3	4	5	6
$P(X=x)$	0.1	$2k$	k	0.2	$3k$	0.1

तो k का मान है ? Then value of k is

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

20. If (यदि) $P(A) = 0.2$, $P(B/A) = 0.3$ then (तो) $P(A \cap B) =$

(a) 0.06 (b) 0.03 (c) 0.02 (d) 0.05

21. The vector $2\hat{i} + \hat{j} - \hat{k}$ is perpendicular to $\hat{i} - 4\hat{j} - \lambda\hat{k}$ then the value of λ is

यदि सदिश $2\hat{i} + \hat{j} - \hat{k}$, $\hat{i} - 4\hat{j} - \lambda\hat{k}$ पर लम्ब है, तो λ का मान है ?

(a) 0 (b) –1 (c) 2 (d) 3

22. Let A = {1, 2} then how many binary operations can be defined on the set A.

यदि A = {1, 2} तो समुच्चय A पर कितने द्विधारी संक्रियाएँ परिभाषित होगी ?

(a) 8 (b) 10 (c) 16 (d) 20

23. Which of the following is the direction cosine of any line which is equally inclined to the axes.

अक्षों पर समान झुकाव वाले किसी रेखा का दिक् कोज्या निम्नलिखित में कौन-सा है ?

(a) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (b) $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ (c) $\left(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{4}{\sqrt{3}}\right)$ (d) None of these (इनमें से कोई नहीं)

24. Which of the following is a scalar matrix ?

निम्न में से कौन एक अदिश आव्युह है।

(a) $\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$ (c) $\begin{vmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix}$ (d) $\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$

25. $l = m = n = 1$ represents the direction cosine of which ones ?

$l = m = n = 1$ किस अक्ष के दिक् कोज्या को प्रदर्शित करता है ?

(a) x-axis (x - अक्ष) (b) y - axis (y - अक्ष)

(c) z - axis (z - अक्ष) (d) None of these (इनमें से कोई नहीं)

26. The value of $\int_0^{\pi^2/4} \sin\sqrt{x}\, dx$ is

($\int_0^{\pi^2/4} \sin\sqrt{x}\, dx$ का मान है ?

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) None of these (इनमें से कोई नहीं)

27. If $\vec{a}\cdot(\vec{b}\times\vec{c})=3$ then which of the following is true ?

(यदि $\vec{a}\cdot(\vec{b}\times\vec{c})=3$, तो निम्नलिखित में कौन सही है ?)

(a) $\vec{c}\cdot(\vec{a}\times\vec{b})=-3$ (b) $\vec{a}\cdot(\vec{c}\times\vec{b})=-3$

(c) $\vec{b}\cdot(\vec{a}\times\vec{c})=3$ (d) $(\vec{a}\times\vec{c})\cdot\vec{b}=3$

28. $\int_0^{\pi} xf(\sin x)\, dx =$

(a) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)\, dx$ (b) $\frac{\pi}{4}\int_0^{\pi} f(\sin x)\, dx$ (c) $\int_0^{\pi/2} f(\sin x)\, dx$ (d) $\frac{\pi}{3}\int_0^{\pi/2} f(\cos x)\, dx$

29. **Which of the following is symmetric and Reflexive but not transitive ?**
(निम्न में से कौन सममित और स्वतुल्य है किन्तु संक्रमक नहीं है ?)
(a) {(1, 1), (2, 3), (3, 1)} (b) {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)}
(c) {(2, 2), (3, 3), (1, 2), (1, 1)} (d) None of these (इनमें से कोई नहीं)

30. **Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$ then total number of relation from A to B will be**
माना कि $A = \{1, 2, 3\}$ और $B = \{a, b\}$ तो A से B पर कुल कितने सम्बन्ध हो सकते हैं ?
(a) 64 (b) 32 (c) 128 (d) 6

31. **If $\int_0^a f(x)dx = 10$ then value of $\int_0^a f(a-x)dx =$**
(यदि $\int_0^a f(x)dx = 10$, तो $\int_0^a f(a-x)dx$ का मान है ?
(a) $\frac{5}{2}$ (b) 10 (c) 6 (d) 5

32. **Two dice are tossed. Probability of getting a sum 7 is ?**
(दो पासों को उछाला जाता है। दोनों पर आने वाले अंकों का योग 7 आने की प्रायिकता है ?)
(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{9}$ (d) $\frac{5}{36}$

33. **If A and B are two symmetric matrix of same order, then which is true.**
यदि A और B दो एक ही कोटि के सममित आव्युह है तो कौन सत्य है ?
(a) $A + B$ is symmetric ($A + B$ सममित है)(b) AB is symmetric ($A\ B$ सममित है)
(c) Both (a) & (b) are true ((a) और (b) दोनों सत्य है ।) (d) None of these (इनमें से कोई नहीं)

34. **The equation $\vec{r} = k_1\hat{i} + k_2\hat{j}$ represents the plane ?**
(समीकरण $\vec{r} = k_1\hat{i} + k_2\hat{j}$, तल को प्रदर्शित करता है ?)
(a) $x = 0$ (b) $z = 0$ (c) $y = 0$ (d) None of these (इनमें से कोई नहीं)

35. **The value of $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}]$ is equal to**
($[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}]$ का मान बराबर है ?)
(a) $[\vec{a}\ \vec{b}\ \vec{c}]^2$ (b) $2[\vec{a}, \vec{b}, \vec{c}]$ (c) $2[\vec{a} + \vec{b} + \vec{c}]$ (d) None of these (इनमें से कोई नहीं)

36. **The projection of the vector $\hat{i} - 2\hat{j} + \hat{k}$ on the vector $2\hat{i} - \hat{j} + \hat{k}$ is**
(सदिश $2\hat{i} - \hat{j} + \hat{k}$ पर सदिश $\hat{i} - 2\hat{j} + \hat{k}$ का प्रक्षेप है ?)
(a) $\frac{4}{\sqrt{6}}$ (b) $\frac{5}{\sqrt{6}}$ (c) $\frac{4}{\sqrt{6}}$ (d) $\frac{7}{\sqrt{6}}$

37. **If $\vec{a}.\vec{b} = 0$, then which is true ?**
(यदि $\vec{a}.\vec{b} = 0$ तो कौन-सा सत्य है ?)

(a) $\vec{a} \perp \vec{b}$ (b) $\vec{a} \parallel \vec{b}$ (c) $\vec{a} + \vec{b} = 0$ (d) $\vec{a} - \vec{b} = 0$

38. If (यदि) $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$ then (तो)

(a) $x = 3, y = 1$ (b) $x = 1, y = 3$ (c) $x = 0, y = 3$ (d) $x = 0, y = 0$

39. If the value of determinant $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$ then which one is true ?

(यदि सारणिक $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$ तो कौन सा एक सत्य है ?

(a) $a = -3$ (b) $a = 2$ (c) $a = 1$ (d) $a = 3$

40. If $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$ the fog equal to

(यदि $f(x) = 8x^3$ और $g(x) = x^{\frac{1}{3}}$ तो fog बराबर है ?)

(a) $3x$ (b) $9x$ (c) $4x$ (d) $8x$

41. Let A and B be 3×3 matrices. Then which of the following is true if $|A - B| = 0$
(माना कि A और B, 3×3 कोटि का आव्युह है । यदि $|A - B| = 0$ तो निम्नलिखित में कौन सा सत्य है ?

(a) $A = 0$ या (or) $B = 0$ (b) $|A| = 0$ and (और) $|B| = 0$
(c) $|A| = 0$ or (या) $|B| = 0$ (d) $A = 0$ and (और) $B = 0$

42. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(अवकल समीकरण $\frac{dy}{dx} = \frac{y}{x}$ का हल है ?)

(a) $y = \frac{k}{x}$ (b) $y = kx$ (c) $y = k \log x$ (d) $\log y = kx$

43. The integrating factor of $x \log \frac{dy}{dx} + y = 2 \log x$ is

($x \log x \frac{dy}{dx} + y = 2 \log x$ का समाकलन गुणांक है?)

(a) x (b) e^x (c) $\log x$ (d) $\log(\log x)$

44. $\int_{-1}^{2} x|x|dx =$

(a) $\int_{-1}^{2} x^2 dx$ (b) $\frac{7}{3}$ (c) 3 (d) None of these (इनमें से कोई नहीं)

45. If (यदि) $x = e^{y+e^{y+e^{y+\cdots\cdots\infty}}}$, $x > 0$, then (तो) $\frac{dy}{dx}$

(a) $\frac{1-x}{x}$ (b) $\frac{1}{x}$ (c) $\frac{x}{1+x}$ (d) $\frac{1+x}{x}$

46. If $f(x) = e^x, x \in [0, 1]$, then a number 'c' of Lagrange's mean value theorem is ?
(यदि $f(x) = e^x, x \in [0, 1]$, तो लैंगराँजेज माध्य मान प्रमेय के लिए 'c' का मान है।

(a) log (e –1) (b) log (e +1) (c) loge (d) None of these (इनमें से कोई नहीं)

47. If $x + y = k$ is normal to $y^2 = 12x$ then value of k is
(यदि $x + y = k$, $y^2 = 12x$ पर अभिलम्ब है तो k का मान है ?)

(a) 3 (b) 9 (c) –9 (d) –3

48. Maximum value of $\frac{\log x}{x}$ in $(2, \infty)$ is

(अन्तराल $(2, \infty)$ में $\frac{\log x}{x}$ का अधिकतम मान है ?)

(a) $\frac{\log 2}{2}$ (b) 0 (c) $\frac{1}{e}$ (d) 1

49. If $\int e^x [f(x) + f'(x)].dx = e^x.\sin x$ the $f(x) =$

(यदि $\int e^x [f(x) + f'(x)].dx = e^x.\sin x$ तो $f(x)$)

(a) $\sin x$ (b) $-\sin x$ (c) $\cos x - \sin x$ (d) $\sin x + \cos x$

50. $\hat{i} \times \hat{j} =$

(a) $\hat{k}$ (b) $-\hat{k}$ (c) $\hat{i}$ (d) $-\hat{i}$

SECTION (खण्ड)–II

Non-Objective Questions (गैर वस्तुनिष्ट प्रश्न)

Time : [2 Hrs] **Full Marks : 50**

समय : 2 घंटा **पूर्णांक : 50**

Question number 1 to 22 are of short Answer type. Each question carry 2 marks. Answer any 15 Question.

[प्रश्न संख्या 1 से 22 तक लघुउत्तरीय प्रकार के है । प्रत्येक के लिए 2 अंक निर्धारित है। किन्हीं 15 प्रश्नों के उत्तर दें)

$15 \times 2 = 30$

1. If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$ then find the angle between $\vec{a}$ & $\vec{b}$

(यदि $\vec{a} + \vec{b} + \vec{c} = 0$ और $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$ तो $\vec{a}$ और $\vec{b}$ के बीच का कोण ज्ञात करें ।)

2. Find maximum value of $\frac{\log x}{x}$

($\frac{\log x}{x}$ का अधिकतम मान ज्ञात करें ।)

3. If (यदि) $x^3 + y^3 = \sin(x + y)$, find (ज्ञात करे) $\frac{dy}{dx}$

4. Test the continuity of the function $f(x) = |x|$ for $x = 0$
(फलन $f(x) = |x|$ के संतता की जाँच $x = 0$ पर करें ।)

5. Six coin are tossed simultaneously. Find the Probability of getting (i) no head (ii) at least one head
छः सिक्को को एक साथ उछाला जाता है । तो (i) कोई शीर्ष नहीं (ii) कम से कम एक शीर्ष, प्राप्त करने की प्रायिकता ज्ञात करें ।

6. Find (ज्ञात करें) $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

7. Prove that (सिद्ध करे)

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

8. Integrate (समाकलन निकाले) : $\int \frac{1}{1-\sin x} dx$

9. Find the equation of tangent and normal to the curve $x^2 + y^2 = 4$ at the point (2, 0)
(वक्र $x^2 + y^2 = 4$ के बिन्दु (2, 0) पर स्पर्शरेखा एवं आभिलम्ब का समीकरण ज्ञात करें ?)

10. If $y = \tan^{-1} \frac{2x}{1-x^2}$, find $\frac{dy}{dx}$

(यदि $y = \tan^{-1} \frac{2x}{1-x^2}$ तो $\frac{dy}{dx}$ ज्ञात करें ।)

11. Form the differential equation corresponding to the curve $y = ae^{bx}$, a and b are arbitarry constant.
(वक्र $y = ae^{bx}$ के संगत अवकल समीकरण ज्ञात करें । जहाँ a और b नियतांक है ।)

12. Find a 2×2 matrix B such that $\begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$

(एक 2×2 कोटि का आव्युह B ज्ञात करें ताकि $\begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$)

13. Let $f : R \to R$ and $g : R \to R$ be two functions such that $fo\,g\,(x) = \sin x^2$ and $go\,f(x) = \sin^2 x$. find $f(x)$ and $g\,(x)$
(माना कि $f : R \to R$ और $g : R \to R$ दो फलन इस प्रकार है कि $fo\,g\,(x) = \sin x^2$ और $go\,f(x) = \sin^2 x$ $f(x)$ और $g(x)$ ज्ञात करें ।)

14. Give example of a relation which is reflexive and transitive but not symmetric.
(एक सम्बन्ध का उदाहरण दे जो स्वतुल्य और संक्रमक है किन्तु सममित नहीं है ?)

15. If P(A) = 0.4, P(B) = P, P(A ∪ B) = 0.6 and A, B are independent events. Find the value of P.
(यदि P(A) = 0.4, P(B) = P, P(A ∪ B) = 0.6 और A, B स्वतंग घटनाएँ हैं । P का मान ज्ञात करें ।)

16. If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-5}$ are perpendicular, find the value of k

(यदि रेखाएँ $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ और $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-5}$ लम्बवत है, तो k का मान ज्ञात करें)

17. The mean and varience of a binomial distribution are 12 and 3 respectively. Find the probability distribution.
(एक द्विपद वितरण का माध्य और प्रसरण क्रमशः 12 और 3 है। प्रायिकता वितरण ज्ञात करें ।)

18. Find co-ordinate of the point where the line through (3, 4, 1) and (5, 1, 6) crosses xy-plane.
(बिन्दु (3, 4, 1) और (5, 1, 6) से गुजरनेवाली रेखा, xy तल के जिस बिन्दु से गुजरती है उसका नियामक ज्ञात करें ।)

19. Evaluate (मान ज्ञात करें) : $\int \tan x \, dx$

20. Show that the vector $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} - 3\hat{j} - 5\hat{k}$ are at right angles.
(दिखाइए कि सदिश $2\hat{i} - \hat{j} + \hat{k}$ और $\hat{i} - 3\hat{j} - 5\hat{k}$ परस्पर लम्बवत है ।)

21. Find the equation of plane which cuts equal intercepts from the axes and passes through the point (2, 3, 5).
(अक्षों को समान दूरी पर काटने और बिन्दु (2, 3, 5) से गुजरने वाली समतल का समीकरण ज्ञात करें।)

22. Solve (हल करें ।)
$(x + \log y)\, dy + y dx = 0$

प्रश्न संख्या 23 से 26 दीर्घउत्तरीय कोटि के है । प्रत्येक के लिए 5 अंक निर्धारित है । प्रत्येक प्रश्न या उसके विकल्प में से किसी एक का उत्तर दें ।

Question number 23 to 26 are of long answer type. Each question carry 5 marks. Answer every question or alternative of it. [4 × 5]

23. Find the area of the region in the first quadrant enclosed by the x-axis, the line $x=\sqrt{3}y$ and the circle $x^2 + y^2 = 4$
x-अक्ष, रेखा $x=\sqrt{3}y$ और वृत्त $x^2 + y^2 = 4$ से घिरे क्षेत्र का क्षेत्रफल प्रथम पाद में ज्ञात करें ।

Or, (या)

The first of three bag contains 7 white & 10 black ball, the second contain 5 white & 12 black ball and the third contains 17 white balls. A person chooses an bag at random and draws a ball from it & finds it to be white. Find the Probability that the ball came from the second bag.
(तीन में से पहले बैग में 7 सफेद और 10 काले गेंद, दूसरे में 5 सफेद और 12 काले गेंद और तीसरे में 17 सफेद गेंद है। एक आदमी एक बैग चुनता है और उसमें से एक गेंद निकालता है और पाता है कि वह सफेद है। निकाले गये गेंद के दूसरे बैग के होने की प्रायिकता ज्ञात करें ।)

24. Prove that (सिद्ध करें) : $\int_0^{\pi/2} \sin 2x . \log \tan x \, dx = 0$

Or, (या)

$\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) \, dx$

25. Find the shortest distance and the vector equation of the line of shortest distance between the lines given by

$\vec{r} = (2\vec{j} - 3\vec{k}) + \lambda (2\vec{i} - \vec{j})$

and $\vec{r} = (4\vec{j} + 3\vec{k}) + \mu (3\vec{i} + \vec{j} + \vec{K})$

दिये गये रेखा $\vec{r} = (2\vec{j} - 3\vec{k}) + \lambda (2\vec{i} - \vec{j})$

और $\vec{r} = (4\vec{i} + 3\vec{k}) + \mu (3\vec{i} + \vec{j} + \vec{K})$

के बीच न्यूनतम दूरी और न्यूनतम दूरी रेखा का सदिश समीकरण ज्ञात करें ?

Or, (या)

Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

(रेखा $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ में बिन्दु (1, 6, 3) का प्रतिबिम्ब ज्ञात करें ।

26. Solve the LPP

Maximize $z = 30x + 25y$

Subjected to $3x + 3y \leq 18$

and $3x + 2y \leq 15$

$x \geq 0, y \geq 0$

LPP को हल करें

अधिकतमीकरण $z = 30x + 25y$

जबकि $3x + 3y \leq 18$

और $3x + 2y \leq 15$

$x \geq 0, y \geq 0$

Or, (या)

In a bank Principal increases at the rate of 5% per year. In how many years Rs. 1000 double itself.

(एक बैंक में मूलधन 5% वार्षिक ब्याज की दर से बढ़ती है। कितने वर्षों में रु॰ 1000 अपने का दुगुना होगा ?)

SOLUTION
Answer Key

Objective Question :

1. (d)	2. (a)	3. (b)	4. (a)	5. (d)	6. (b)	7. (d)	8. (a)	9. (b)	10. (c)
11. (a)	12. (d)	13. (a)	14. (c)	15. (a)	16. (d)	17. (d)	18. (a)	19. (a)	20. (a)
21. (c)	22. (c)	23. (a)	24. (c)	25. (d)	26. (d)	27. (b)	28. (a)	29. (b)	30. (a)
31. (b)	32. (a)	33. (a)	34. (b)	35. (b)	36. (b)	37. (a)	38. (d)	39. (a)	40. (d)
41. (d)	42. (b)	43. (c)	44. (b)	45. (d)	46. (a)	47. (b)	48. (c)	49. (a)	50. (c)

Solution to Short Answer type question :

1. Let 'θ' be the angle between $\vec{a}$ and $\vec{b}$

Since, $\vec{a}+\vec{b}+\vec{c}=0$ $\Rightarrow \vec{a}+\vec{b}=-\vec{c}$

Now, $|\vec{a}+\vec{b}|^2=(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}$

So, $|\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}$

$\Rightarrow 7^2=3^2+5^2+2\vec{a}.\vec{b}$ $\Rightarrow 49-34=15=2\vec{a}.\vec{b}$

$\Rightarrow 2|\vec{a}|.|\vec{b}|\cos\theta=15$ $\Rightarrow 2\times3\times5\cos\theta=15$

$\Rightarrow 2\cos\theta=1 \Rightarrow \cos\theta=\frac{1}{2}$

$\therefore \theta=\frac{\pi}{3}$

2. Let $y=\frac{\log x}{x}$

$$\therefore \frac{dy}{dx}=\frac{1}{x}.\frac{1}{x}+\log x\left(-\frac{1}{x^2}\right)=\frac{1-\log x}{x^2} \quad ...(1)$$

$$\text{Again, } \frac{d^2y}{dx^2}=\frac{x^2\left(-\frac{1}{x}\right)-(1-\log x).2x}{x^4}$$

$$=\frac{-x-2x+2x\log x}{x^4}=\frac{2x\log x-3x}{x^4}$$

for maxima & minima

Now, Put $\frac{dy}{dx}=0 \Rightarrow 1-\log x=0 \Rightarrow \log x=1=\log e \Rightarrow x=e$

At $x = e$, $\frac{d^2y}{dx^2} = \frac{2e\log e - 3e}{e^4} = \frac{2e-3e}{e^4} = -\frac{1}{e^3}$ (–ve)

So, at $x = e$, y has maximum value

$\therefore$ Maximum of $\frac{\log x}{x} = \frac{\log e}{e} = \frac{1}{e}$

3. Given that,

$x^3 + y^3 = \sin(x + y)$

Differentiating w.r.t. 'x', we have

$$\frac{d}{dx}(x^3+y^3) = \frac{d}{dx}\sin(x+y)$$

$$\Rightarrow \frac{dx^3}{dx} + \frac{dy^3}{dy} = \frac{d\sin(x+y)}{d(x+y)} \cdot \frac{d(x+y)}{dx}$$

$$\Rightarrow 3x^2 + 3y^2 \cdot \frac{dy}{dx} = \cos(x+y)\left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow [3y^2 - \cos(x+y)] \cdot \frac{dy}{dx} = \cos(x+y) - 3x^2$$

$$\therefore \frac{dy}{dx} = \frac{\cos(x+y) - 3x^2}{3y^2 - \cos(x+y)}$$

4. Given function $f(x) = |x|$ $= x, x > 0$

$= -x, x < 0$

$= 0, x = 0$

Hence $f(0) = 0$

Now, Left hand limit $= \lim_{x \to 0-} f(x) = \lim_{x \to 0} (-x) = 0$ $[\because f(x) = -x, x < 0]$

Right hand limit $= \lim_{x \to 0+} f(x) = \lim_{x \to 0} (x) = 0$

So, LH: = RHL = $f(0)$

Hence, $f(x)$ is continuous at $x = 0$

[This can also be verified by graph of $|x|$]

5. Let P = Probability of getting a head in the single toss of coin

$= \frac{1}{2}$

and q = Probability of not getting a head

$= 1 - P = 1 - \frac{1}{2} = \frac{1}{2}$

Let Number of successes in the experiment be '×'

So, × can take the value 0, 1, 2, 3, 4, 5, 6

Also, x = no. of trial = 6

(i) P (no head) = P($x = 0$) = ${}^6C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{6-0} = \frac{1}{64}$

(ii) P (at least one head) = 1 – P (no bead) = $1-\frac{1}{64}=\frac{63}{64}$

6. Let I = $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\frac{\sin x}{\cos x}}} = \int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$

Also, I = $\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \quad \left[\because \int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx\right]$ ...(1)

By (1) + (2) $\Rightarrow$ 2I = $\int_{\pi/6}^{\pi/3}dx=[x]_{\pi/6}^{\pi/6}=\left(\frac{\pi}{3}-\frac{\pi}{6}\right)=\frac{\pi}{6}$

$\therefore$ I = $\frac{\pi}{12}$

7. Ler Δ = $\begin{vmatrix}1 & a & a^3\\1 & b & b^3\\1 & c & c^3\end{vmatrix} = \begin{vmatrix}1 & a & a^3\\0 & b-a & b^3-a^3\\0 & c-a & c^3-a^3\end{vmatrix}$ [By $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$]

= (b – a) (c – a) $\begin{vmatrix}1 & a & a^3\\0 & 1 & a^2+ab+a^2\\0 & 1 & c^2+ca+a^2\end{vmatrix}$ (taking common (b –a) from R_2 and (c –a) from R_3)

= (b –a) (c –a) .1. $\begin{vmatrix}1 & b^2+ab+a^2\\1 & c^2+ca+a^2\end{vmatrix}$ (expanding along c_1)

= $(b-a)\,(c-a)\,[c^2+ca+a^2-(b^2+ba+a^2)]$

= $(b-a)\,(c-a)\,[(c^2-b^2)+a\,(c-b)]$

$\Rightarrow$ $(a-b)\,(b-c)\,(c-a)\;(a+b+c)$

8. Let I = $\int\frac{1}{1-\sin x}dx = \int\frac{1}{1-\sin x}\times\frac{1+\sin x}{1+\sin x}dx$

(Multiplying numerator & denominator by 1 + sin$x$)

= $\int\frac{1+\sin x}{1-\sin^2 x}dx = \int\frac{1+\sin x}{\cos^2 x}.dx$

= $\int\left\{\frac{1}{\cos^2 x}+\frac{\sin x}{\cos^2 x}\right\}.dx = \int(\sec^2 x+\sec x.\tan x)dx$

= $\int\sec^2 x\,dx+\int\sec x.\tan x\,dx=\tan x+\sec x+k$

9. Given curve is $x^2 + y^2 = 4$

Differentiating both side w.r.t. x, we have

$$2x + 2y.\frac{dy}{dx} = 0 \qquad \Rightarrow y.\frac{dy}{dx} = -x$$

$$\therefore \quad \frac{dy}{dx} = -\frac{x}{y}$$

At, (2, 0), $\frac{dy}{dx} = \frac{-2}{0} = \infty$ (not defined)

i.e., tangent are parallel to y -axis
So, Equation of tangent is $x = 2$
and equation of normal is $y = 0$

[If at P(α, β) $\frac{dy}{dx}$ is not defined than equation of tangent is $x = \alpha$ & equation of normal is $y = \beta$]

10. Here, $y = \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x$ $\qquad \left[\because \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x\right]$

Differentiating w.r.t. 'x', we have

$$\frac{dy}{dx} = 2\,\frac{d(\tan^{-1}x)}{dx} = 2\times\frac{1}{1+x^2}$$

11. Given, $y = ae^{bx}$ ... (1)

Differentiating (1) w.r.t 'x' we get

$$\frac{dy}{dx} = abe^{bx} = b.\,ae^{bx} = by \qquad ...(2)$$

Again, from (2)

$$\frac{d^2y}{dx^2} = b.\frac{dy}{dx} = \frac{1}{y}.\frac{dy}{dx}.\frac{dy}{dx} \quad \left[\text{By (2) } b = \frac{1}{y}.\frac{dy}{dx}\right]$$

$$= \frac{1}{y}.\left(\frac{dy}{dx}\right)^2$$

$$\therefore \quad y.\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

Which is require differential Equation.

12. Let $A = \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$, $C = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$

Let B be a 2×2 matrix such that $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given $AB = C$

So, $\begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}.\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 6a+5c & 6b+5d \\ 5a+6c & 5b+6d \end{bmatrix} = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$$

$$\Rightarrow \begin{cases} 6a + 5c = 11, 6b + 5d = 0 \\ 5a + 6c = 0, 5b + 6d = 0 \end{cases}$$

Solving , we get—

$a = 6, c = -5, b = -5, d = 6$

Hence, $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} 6 & -5 \\ -5 & 6 \end{vmatrix}$

13. Given, $(f \text{ og})(x) = \sin x^2$ and $(gof)(x) = \sin^2 x$

$\Rightarrow f(g(x)) = \sin(x^2)$ & $g(f(x) = (\sin x)^2$ ...(1)

Clearly, $f(x) = \sin x$ & $g(x) = x^2$ satisfies both conditions of (1)

So, $f(x) = \sin x$ and $f(x) = x^2$

14. Let $A = \{1, 2, 3\}$ and R be a relation on A.

Consider, $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$

Then the relation R is Reflecive and Transitive but not symmetric.

15. Since A and B are independent events

$\therefore \quad P(A \cap B) = P(A).P(B)$

Now, $P(A \cup B) = P(A) + P(B) - (P(A \cap B)$

$= P(A) + P(B) - P(A).P(B)$

$\Rightarrow \quad 0.6 = 0.4 + P - 0.4P$

$\Rightarrow \quad 0.6P = 0.2 \therefore P = \frac{0.2}{0.6} = \frac{1}{3}$

16. The direction Ratio of two lines are given by $(a_1, b_1, c_1) = (-3, 2k, 2)$ and $(a_2, b_2, c_2) = (3k, 1, -5)$

If the two lines are perpendicular than $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow (-3)(3k) + (2k)1 + (2)(-5) = 0$

$\Rightarrow -9k + 2k - 10 = 0 \Rightarrow -7k - 10 = 0 \Rightarrow 7k = -10$

$\therefore \quad k = -\frac{10}{7}$

17. For binomial distribution,

mean $= nP = 12$ and varience $= npq = 3$

$\Rightarrow 12q = 3 \quad \Rightarrow q = \frac{1}{4}$

$\therefore \quad P = 1 - q = 1 - \frac{1}{4} = \frac{3}{4}$

and $n.p = 12 \Rightarrow n.\frac{3}{4} = 12 \Rightarrow n = 16$

$\therefore$ Binomial distribution is

$$(P + q)^n = \left(\frac{3}{4} + \frac{1}{4}\right)^{16}$$

18. Let $A = (3, 4, 1)$ and $B = (5, 1, 6)$

Then direction ratio of line AB are $3 - 5, 4 - 1, 1 - 6 = (-2, 3, 5)$

So, Equation of any line AB will be

$$\frac{x-3}{-2} = \frac{y-4}{3} = \frac{z-1}{-5} = r \text{ (Let)}$$

Co-ordinate of any point P of above line is

$p(-2r+3, 3r+4, -5r+1)$

If p lies on xy plane, then $-5r+1=0$, $\Rightarrow r=\frac{1}{5}$

$\therefore$ co-ordinate require is $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$

19. $I = \int \tan x\, dx = \int \frac{\sec x . \tan x}{\sec x} . dx$

Put $z = \sec x \Rightarrow dz = \sec x \tan x\, dx$

$\therefore$ $I = \int \frac{dz}{z} = \log|z| + c = \log|\sec x| + c$

20. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$

Now, $|\vec{a}| = \sqrt{2^2+(-1)^2+1^2} = \sqrt{6}$ and $|\vec{b}| = \sqrt{1^2+(-3)^2+(-5)^2} = \sqrt{35}$

$\therefore$ $\vec{a} . \vec{b} = 2 \times 1 + (-1) \times (-3) + 1(-5) = 0$

Hence, given vectors are at right angles.

21. Let the plane cuts intercept of 'a' unit on axes.

So, equation of the plane be

$\frac{x}{a} + \frac{y}{a} + \frac{z}{a} = 1 \Rightarrow x + y + x = a$

If this plane passes through the point $P(2, 3, 5)$ then $2 + 3 + 5 = a \Rightarrow a = 10$

Hence, require equation of plane is $x + y + z = 10$

22. Given differential equation can be be written as $(xdy + ydx) + \log y.\, dy = 0$

Or, $d(xy) + \log y dy = 0$

Integrating we get

$\int d(xy) + \int \log y\, dy = c$

$\Rightarrow$ $xy + y\log y - y = c$

which is the required solution.

Long Answer Type Question (दीर्घ उत्तरीय प्रश्न)

23. The given circle is $x^2 + y^2 = 4$ ...(1)

Its centre is (0, 0) and radius is 2.

Given line is $x = \sqrt{3}y$ ...(2)

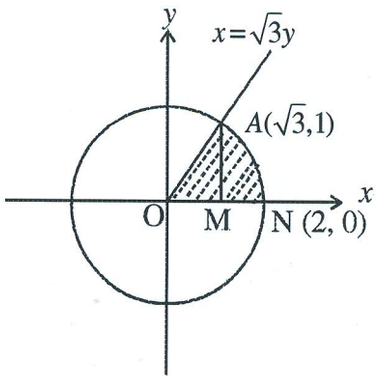
solving (1) & (2) to get point of intersection

$(\sqrt{3}y)^2 + y^2 = 4 \Rightarrow 4y^2 = 4 \Rightarrow y^2 = 1$

$\Rightarrow$ $y = 1, -1$ But $y \geq 0$ for first quadrant.

$\therefore$ for $y = 1$, $x = \sqrt{3}.1 = \sqrt{3}$

So, point of intersection is $(\sqrt{3}, 1)$



Now, the require area = area of shaded region

$$= \text{area (OAMO)} + \text{area (MANM)}$$

$$= \int_0^{\sqrt{3}} y\,dx + \int_{\sqrt{3}}^{2} y\,dx$$

$$= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}\,dx + \int_{\sqrt{3}}^{2} \sqrt{4-x^2}.dx$$

$$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{\sqrt{3}}\left(\frac{3}{2}-0\right) + \frac{1}{2}(0-\sqrt{3}) + 2\left[\sin^{-1}1 - \sin^{-1}\frac{\sqrt{3}}{2}\right]$$

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + 2\left(\frac{\pi}{2}-\frac{\pi}{3}\right) = \frac{\pi}{3} \text{ sq. unit.}$$

Or,

Let A denote the event of drawing a white ball, when one ball is drown at random from one of the three urns.

Let A_1 = Event that ball drawn is from the second urn.

A_2 = Event that ball drawn is from the first urn.

A_3 = Event that ball drawn is from third urn.

By Bayes' theorem,

$$P(A_1/A) = \frac{P(A_1).P(A/A_1)}{P(A_1).P(A/A_1)+P(A_2).P(A/A_2)+P(A_3).P(A/A_3)}$$

Here, $P(A/A_1)$ = Prob. of drawing a white ball when the ball is drawn from second urn.

$P(A/A_2) = \frac{7}{17}$, $P(A/A_3) = \frac{17}{17}$

Also, prob. of choosing first, second and third urns are assumed to be equal

So, $P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$

$$\therefore\ P(A_1/A) = \frac{\frac{1}{3}.\frac{5}{17}}{\frac{1}{3}.\frac{5}{17}+\frac{1}{3}.\frac{7}{17}+\frac{1}{3}.\frac{17}{17}} = \frac{5}{29}$$

24. Let $I = \int_0^{\pi/2} \sin 2x.\log\tan x\,dx$ ... (1)

Then, $I = \int_0^{\pi/2} \sin 2\left(\frac{\pi}{2}-x\right).\log\tan\left(\frac{\pi}{2}-x\right)dx$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\pi/2} \sin 2x . \log \cot x \, dx \qquad ...(2)$$

By (1) + (2)

$$2I = \int_0^{\pi/2} \sin 2x \{\log \tan x + \log \cot x\} dx$$

$$= \int_0^{\pi/2} \sin 2x . \log(\tan x . \cot x) . \, dx$$

$$= \int_0^{\pi/2} \sin 2x . \log 1 \, dx = 0$$

$\therefore$ I = 0

Or,

$$\text{Let I} = \int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) \, dx$$

$$= \int_0^{\pi/2} \{2 \log \sin x - \log(2 \sin x . \cos x\} dx$$

$$= \int_0^{\pi/2} \{2 \log \sin x - \log 2 - \log \sin x - \log \cos x\} dx$$

$$= \int_0^{\pi/2} (\log \sin x - \log 2 - \log \cos x) \, dx$$

$$= \int_0^{\pi/2} (\log \sin x \, dx - \int_0^{\pi/2} \log 2 \, dx - \int_0^{\pi/2} \log \cos x \, dx$$

$$= \int_0^{\pi/2} \log \sin x \, dx - \log 2 \int_0^{\pi/2} dx - \int_0^{\pi/2} \log \cos\left(\frac{\pi}{2} - x\right) dx$$

$$\left[\because \int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx\right]$$

$$= \int_0^{\pi/2} \log \sin x \, dx - \log 2 [x]_0^{\pi/2} - \int_0^{\pi/2} \log \sin x . dx$$

$$= -\log 2 \left(\frac{\pi}{2} - 0\right)$$

$$\therefore \text{ I } = -\frac{\pi}{2} \log 2$$

25. The Equation of given line in cartesian form can be written as

$$\frac{x-0}{2}=\frac{y-2}{-1}=\frac{z+2}{0}=\lambda \qquad ...(1)$$

and $\frac{x-4}{3}=\frac{y-0}{1}=\frac{z-3}{1}=\mu$ ... (ii)

Line (i) passes through point (0, 2, – 3) and d.r. is (2, –1, 0), line (ii) passes through point (4, 0, 3) and d.r. is (3, 1, 1). From the given vector form, we can find variable point on each line, say

P(2λ, 2–λ , –3) on l_1 and Q (4 + 3μ, μ, 3 + μ) on l_2

The d.r. of PQ is (4+ 3 μ –2λ, μ – 2 + λ, 3 + μ + 3)

If $\vec{PQ}$ is taken as the shortest distance vector, then it should be perpendicular to both l_1 & l_2

So, (4 + 3μ –2λ). 2 + (μ – 2 + λ) (–1) + (6 + μ) . 0 = 0 ...(iii)

and (4 + 3μ – 2λ). 3 + (μ –2 + λ). 1 + (6 + μ)0 . = 0 ...(iv)

By (iii), (4 + 3μ – 2λ). 2 – (μ – 2 + λ) = 0

⇒ 8 + 6 μ – 4λ – μ + 2 – λ = 0

⇒ λ – μ = 2 (v)

By (iv), (12 + 9μ – 6λ) + (μ – 2 + λ) + (6 + μ) = 0

⇒ 16 + 11μ – 5 λ = 0 ⇒ 5λ – 11μ = 16 ...(vi)

Solving (v) & (vi), we get, λ = 1 & μ = – 1

Thus point P & Q are P (2, 1, –3) and Q (1, –1, 2)

Hence, Shortest distance $=PQ = \sqrt{(2-1)^2+(1+1)^2+(-3-2)^2}$

$$= \sqrt{1^2+2^2+5^2}=\sqrt{30}$$

Also, vector equation of the shortest distance PQ is

$\vec{r}$ = (Position vector of P) + $t.\ PQ$

$$= (2\hat{i}+\hat{j}-3\hat{k}) + t.(-\hat{i}-2\hat{j}+5\hat{k})$$

Or,

Let given line be AB whose equation is

$$\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3} = k \text{ (let)}$$

Any point on line AB is Q (k, 2k + 1, 3k + 2)

Direction Ratio of line QP are

$k-1, 2k-5, 3k-1$

Since, PQ ⊥ AB

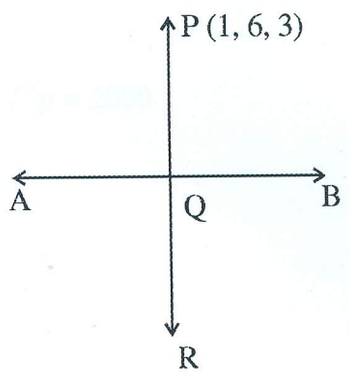
∴ $(k-1)\times 1 + (2k-5)\times 2 + (3k-1)\times 3 = 0$

⇒ $14k - 14 = 0 \Rightarrow k = 1$

So, co-ordinate of Q are (1, 3, 5)

Let R be the image of $P(1, 6, 3)$ *in* AB

Then, Q will be the mid point of PR



$$\therefore \frac{\alpha+1}{2}=1, \frac{\beta+6}{2}=3, \frac{\gamma+3}{2}=5$$

$\Rightarrow \alpha = 1, \beta = 0, \gamma = 7$

Hence, Require image is (1, 0, 7)

26. The given inequalities can be written in the form of equation as—

$3x + 3y = 18$ ... (i)

and $3x + 2y = 15$ ...(ii)

First we draw the line of above equation.

These lines intersect at $P(3, 3)$.

The feasible region is OCPBO which is bounded.

The vertices of the feasible region are: O(0, 0), C(5, 0), P(3, 3) & B(0, 6)

Given, $Z = 30x + 25y$

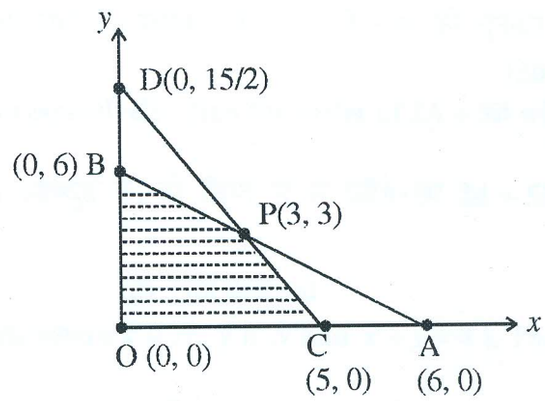
At O(0, 0), $Z = 0$

At C(5, 0), $Z = 150$

At P(3, 3), $Z = 165$

At B(0, 6) $Z = 150$

So, The maximum profit is Rs. 165.



Or,

Let, P be the principal at any time t

Then, $\frac{dp}{dt} = 5\%$ of $p = \frac{5p}{100}$

$\Rightarrow \frac{dp}{dt} = \frac{p}{20}$

$\Rightarrow \frac{1}{p}.dp = \frac{1}{20}dt$

Integrating, we get

$\int \frac{1}{p}dp = \frac{1}{20}\int dt$

$\Rightarrow \log p = \frac{1}{20}t + \log c$ ($\log c$ is arbitrary constant)

$\Rightarrow \log \frac{p}{c} = \frac{1}{20}t$

$\therefore p = ce^{t/20}$ ...(1)

when $t = 0, p = 1000$

By (i), we get $c = 1000$

$\therefore p = 1000\, e^{t/20}$ ...(2)

Let T years be the require time to double the principal. *i.e.* $t = T, p = 2000$

So, By (2), $2000 = 1000\ e^{T/20}$

$\Rightarrow e^{T/20} = 2 \Rightarrow \frac{T}{20} = \log e^2 \Rightarrow T = 20.\log 2$

Hence, principal double in 20 log2 years.