

**SET (प्रारूप)–3**  
**SECTION (खण्ड)–I**

**Objective Questions (वस्तुनिष्ठ प्रश्न)**

**Time : [1 Hrs + 15 Min. (Extra)]**

**Full Marks : 50**

**समय : 1 घंटा + 15 मिनट (अतिरिक्त)**

**पूर्णांक : 50**

There are 50 objective (one correct answer) question carrying one mark each. Choose the correct answer from the given option.

[इस खण्ड में 50 वस्तुनिष्ठ प्रश्न है। प्रत्येक प्रश्न के लिए 1 अंक निर्धारित है। दिए गये विकल्पों में से सही उत्तर चुनें। [50 × 1]

1.  $\int \frac{k^{\sqrt{x}}}{\sqrt{x}} dx =$

- (a)  $k^{\sqrt{x}} \log e^k + c$    (b)  $2k^{\sqrt{x}} \log e^k + c$    (c)  $2k^{\sqrt{x}} \log 10^k + c$    (d)  $2k^{\sqrt{x}} / \log e^k + c$

2.  $\int \frac{1}{\sin x + \cos x} dx =$

- (a)  $\frac{1}{\sqrt{2}} \log \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + k$    (b)  $\log \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + k$   
 (c)  $\frac{1}{2} \log \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + k$    (d) None of these (इनमें से कोई नहीं)

3. Distance between the planes  $\hat{r} \cdot \hat{n} = P_1$  and  $\hat{r} \cdot \hat{n} = P_2$  is

(तल  $\hat{r} \cdot \hat{n} = P_1$  और  $\hat{r} \cdot \hat{n} = P_2$  के बीच की दूरी है ?)

- (a)  $P_1 - P_2$    (b)  $|P_1 - P_2|$    (c)  $\frac{|P_1 - P_2|}{2}$    (d) None of these (इनमें से कोई नहीं)

4. The degree and order of differential equation  $\left(\frac{d^2y}{dx^2}\right)^2 = \left(y + \frac{dy}{dx}\right)^{\frac{1}{2}}$  is which of the following ?

(अवकल समीकरण  $\left(\frac{d^2y}{dx^2}\right)^2 = \left(y + \frac{dy}{dx}\right)^{\frac{1}{2}}$  का घात और कोटि निम्न में से कौन है)

- (a) 4, 2   (b) 2, 4   (c) 3, 4   (d) 4, 3

5. The vector  $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  and  $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  are perpendicular to each other if—

(सदिश  $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  और  $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  एक दूसरे के लम्बवत है यदि—)

- (a)  $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$    (b)  $a_1b_1 + a_2b_2 + a_3b_3$   
 (c)  $a_1b_2 + b_2a_1 + a_3b_3 = 0$    (d) None of these (इनमें से कोई नहीं)

(1)

6. If I be a unit matrix then which is true ?

(यदि I एक एकांकी आव्युह है तो कौन सत्य है?)

- (a)  $I^2 = 1$       (b)  $|I| = 0$       (c)  $|I| = 2$       (d) None of these (इनमें से कोई नहीं)

7.  $\frac{d}{dx}(\sec^{-1} x) =$

- (a)  $\frac{1}{x\sqrt{x^2 - 1}}$       (b)  $\frac{1}{1+x^2}$       (c)  $-\frac{1}{1+x^2}$       (d) None of these (इनमें से कोई नहीं)

8. The equation of normal to the curve  $y = \sin x$  at  $(0, 0)$  is—

(वक्र  $y = \sin x$  के बिन्दु  $(0, 0)$  पर अभिलम्ब का समीकरण है?)

- (a)  $x = 0$       (b)  $y = 0$       (c)  $x + y = 0$       (d)  $x - y = 0$

9.  $\int \tan^2 \frac{x}{2} dx =$

- (a)  $\tan \frac{x}{2} - x + c$       (b)  $\tan \frac{x}{2} + x + c$       (c)  $2 \tan \frac{x}{2} + x + c$       (d)  $2 \tan \frac{x}{2} - x + c$

10. If  $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ , then  $\text{adj}(A) =$

(यदि  $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ , तो  $\text{adj}(A) =$ )

- (a)  $\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$       (b)  $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$       (c)  $\begin{bmatrix} 1 & -2 \\ -2 & -1 \end{bmatrix}$       (d)  $\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$

11. The mean and variance of a binomial distribution are 6 & 4 respectively. The value of parameter  $n$  is—

(एक द्विपद बंटन के माध्य और प्रसरण क्रमशः 6 और 4 हैं; तो स्थिर राशि (प्राचल)  $n$  का मान है?)

- (a) 18      (b) 12      (c) 10      (d) 9

12.  $\begin{vmatrix} \sin 30^\circ & \cos 30^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{vmatrix} =$

- (a) 1      (b) 0      (c)  $\frac{3}{2}$       (d)  $\frac{1}{2}$

13. Value of the determinant is  $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$  is—

(सारणिक  $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$  का मान है?)

- (a) 0      (b)  $256x^3$       (c)  $256x$       (d)  $256x^2$

14. If  $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1} a$  then value of 'a' will be ?

(यदि  $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1} a$ , तो 'a' का मान होगा ?

- (a)  $\frac{1}{4}$       (b)  $\frac{1}{2}$       (c)  $\frac{3}{4}$       (d) 1  
(2)

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15.  $\sin(\tan^{-1}x), (|x| < 1) =$

- (a)  $\frac{x}{\sqrt{1-x^2}}$       (b)  $\frac{1}{\sqrt{1-x^2}}$       (c)  $\frac{1}{\sqrt{1+x^2}}$       (d)  $\frac{x}{\sqrt{1+x^2}}$

16. Let  $R$  be a relation defined as  $\alpha RB$  if  $\alpha$  is perpendicular to  $\beta$ , Where  $\alpha, \beta$  are straight line in a plane, then relation  $R$  is—

(माना कि  $R$  एक सम्बन्ध इस प्रकार है कि  $\alpha RB$  यदि  $\alpha, \beta$  पर लम्ब है जहाँ  $\alpha, \beta$  एक तल में सरल रेखा है तो सम्बन्ध  $R$  है ?)

- (a) reflexive (स्वतुल्य)      (b) symmetric (सममित)      (c) Transitive (संक्रमक)      (d) None of these (इनमें से कोई नहीं)

17. Let  $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$  be a relation on a set  $A = \{1, 2, 3, 4\}$ . The relation  $R$  will be—

(माना कि  $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$  समुच्चय  $A = \{1, 2, 3, 4\}$ . पर एक सम्बन्ध है तो सम्बन्ध  $R$  होगा?)

- (a) Function (फलन)      (b) Reflexive (स्वतुल्य)      (c) symmetric (सममित)      (d) None of these (इनमें से कोई नहीं)

18.  $\hat{k} \times \hat{k} =$

- (a) 0      (b) 1      (c)  $|k|^2$       (d) None of these (इनमें से कोई नहीं)

19. The scalar product of the vector  $5\hat{i} + \hat{j} - 3\hat{k}$  and  $3\hat{i} - 4\hat{j} + 7\hat{k}$  is—

(सदिश  $5\hat{i} + \hat{j} - 3\hat{k}$  और  $3\hat{i} - 4\hat{j} + 7\hat{k}$  का अदिश गुणनफल है ?)

- (a) 10      (b) -10      (c) 15      (d) -15

20.  $\frac{d}{dx} \int f(x) dx =$

- (a)  $f'(x)$       (b)  $f(x)$       (c)  $f''(x)$       (d)  $f(x) + c$

21. If  $A$  is a square matrix such that  $A^2 = A$  then  $(1+A)^3 - 7A$  is equal to—

(यदि  $A$  एक वर्ग आव्युह है कि  $A^2 = A$  तो  $(1+A)^3 - 7A$  का मान है ?)

- (a)  $A$       (b)  $1-A$       (c) 1      (d)  $3A$

22. If (यदि)  $P(A) = 0.8$ ,  $P(B) = 0.5$  and (और)  $P\left(\frac{B}{A}\right) = 0.4$  then (तो)  $P\left(\frac{A}{B}\right) =$

- (a) 0.32      (b) 0.64      (c) 0.16      (d) 0.25

23.  $\int_0^1 \frac{f(x)}{f(x)+f(1-x)} dx =$

- (a) 0      (b)  $\frac{1}{2}$       (c) 1      (d) None of these (इनमें से कोई नहीं)

24.  $\int_{-\pi/2}^{\pi/2} \sin^5 x dx =$

- (a) 0      (b)  $\frac{\pi}{2}$       (c) 1      (d)  $\pi$

(3)

25. The direction ratio of normal to the plane  $7x + 4y - 2z + 5 = 0$  are which of the following ?  
 ( तल  $7x + 4y - 2z + 5 = 0$  पर अभिलम्ब का दिक् कोण्या है ? )
- (a)  $(7, 4, 5)$       (b)  $(7, 4, -2)$       (c)  $(7, 4, 2)$       (d)  $(0, 0, 0)$
26. The angle between the lines  $\vec{r} = (4\hat{i} - \hat{j}) + s(2\hat{i} + \hat{j} - 3\hat{k})$  and  $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + t(\hat{i} - 3\hat{j} + 2\hat{k})$  is—  
 ( रेखा  $\vec{r} = (4\hat{i} - \hat{j}) + s(2\hat{i} + \hat{j} - 3\hat{k})$  और  $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + t(\hat{i} - 3\hat{j} + 2\hat{k})$  के बीच का कोण है ? )
- (a)  $\frac{3\pi}{2}$       (b)  $\frac{\pi}{3}$       (c)  $\frac{2\pi}{3}$       (d)  $\frac{\pi}{6}$
27. The vector equation of the line through the point  $A(3, 4, -7)$  and  $B(1, -1, 6)$  is which of the following ?  
 ( बिन्दु  $A(3, 4, -7)$  और  $B(1, -1, 6)$  से गुजरने वाली रेखा का सदिश समीकरण निम्न में से कौन है ? )
- (a)  $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(\hat{i} - \hat{j} + 6\hat{k})$       (b)  $\vec{r} = (\hat{i} - \hat{j} + 6\hat{k}) + \lambda(3\hat{i} - 4\hat{j} + 7\hat{k})$   
 (c)  $\vec{r} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$       (d) None of these ( इनमें से कोई नहीं )
28. Consider the binary operation \* on  $Q$  defined by  $x * y = 1 + 12x + xy, \forall x, y \in R$  then  $2 * 3$  equals—  
 (  $x * y = 1 + 12x + xy, \forall x, y \in R$  द्वारा परिभाषित  $Q$  पर एक द्विआधारी संक्रिया \* पर विचार कीजिए तो  $2 * 3$  बराबर है ? )
- (a) 31      (b) 41      (c) 43      (d) 51
29. The Range of the function  $f(x) = \sqrt{(x-1)(3-x)}$  is  
 ( फलन  $f(x) = \sqrt{(x-1)(3-x)}$  का परास है ? )
- (a)  $(1, 3)$       (b)  $(0, 1)$       (c)  $(-2, 2)$       (d) None of these ( इनमें से कोई नहीं )
30. If  $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{3}$ , then the value of  $\cos^{-1} x + \cos^{-1} y$  is—  
 ( यदि  $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{3}$  तो  $\cos^{-1} x + \cos^{-1} y$  का मान है ? )
- (a)  $\frac{\pi}{6}$       (b)  $\frac{\pi}{3}$       (c)  $\frac{2\pi}{3}$       (d)  $\pi$
31.  $\int_0^{\frac{\pi}{4}} (\sqrt{\tan x} + \sqrt{\cot x}) dx =$   
 (a)  $\frac{\pi}{\sqrt{2}}$       (b)  $\frac{\pi}{2\sqrt{2}}$       (c)  $\frac{\pi}{4}\sqrt{2}$       (d) None of these ( इनमें से कोई नहीं )
32. The plane  $x = 0$  and  $y = 0$  is—  
 ( तल  $x = 0$  और  $y = 0$  है ? )
- (a) Parallel (समानान्तर)      (b) Perpendicular to each other (एक दूसरे पर लम्ब)  
 (c) Intersect at z-axis (z-अक्ष पर प्रतिच्छेदी)      (d) None of these ( इनमें से कोई नहीं )

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33. For Which value of  $x$ ,  $x(\hat{i} + \hat{j} + \hat{k})$  is a unit vector.

( $x$  के किस मान के लिए  $x(\hat{i} + \hat{j} + \hat{k})$  एक इकाई सदिश है ?

- (a)  $\frac{1}{\sqrt{2}}$       (b)  $\frac{1}{\sqrt{3}}$       (c)  $\pm \frac{1}{\sqrt{3}}$       (d) None of these (इनमें से कोई नहीं)

34.  $\int \frac{1}{x^{\frac{1}{3}}} dx =$

- (a)  $\frac{3}{2}x^{2/3} + c$       (b)  $\frac{2}{3}x^{2/3} + c$       (c)  $\frac{2}{3}x^{-2/3} + c$       (d) None of these (इनमें से कोई नहीं)

35. The problem associated with LPP is—

(रैखिक प्रोग्राम के साथ संबंधित समस्या है ?)

- (a) Single objective function (एक उद्देश्य फलन)  
 (b) Double objective function (दो उद्देश्य फलन)  
 (c) No any objective function कोई उद्देश्य फलन नहीं)  
 (d) None of these (इनमें से कोई नहीं)

36.  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x \cos x + \tan^5 x + 1) dx =$

- (a)  $\frac{\pi}{2}$       (b)  $\pi$       (c) 0      (d) 2

37. If  $\sqrt{x} + \sqrt{y} = 5$  then  $\frac{dy}{dx}$  at (4, 9) is

(यदि  $\sqrt{x} + \sqrt{y} = 5$  तो (4, 9) पर  $\frac{dy}{dx} =$ )

- (a)  $\frac{2}{3}$       (b)  $\frac{3}{2}$       (c)  $-\frac{3}{2}$       (d)  $-\frac{2}{3}$

38. The general solution of the differential equation  $\frac{ydx - xdy}{y} = 0$  is—

(अवकल समीकरण  $\frac{ydx - xdy}{y} = 0$  का व्यापक हल है ?)

- (a)  $xy = c$       (b)  $x = cy^2$       (c)  $y = cx$       (d)  $y = cx^2$

39. If (यदि)  $x > a$ , then (तो)  $\int \frac{dx}{x^2 - a^2} dx =$

- (a)  $\frac{1}{2a} \log \frac{x-a}{x+a} + k$       (b)  $\frac{1}{2a} \log \frac{x+a}{x-a} + k$       (c)  $\frac{1}{a} \log (x^2 - a^2) + k$       (d)  $\log (x + \sqrt{x^2 - a^2}) + k$

40. The mean and variance of a random variable  $X$  are 4 & 2 respectively, then  $P(x = 1)$  is  
 (एक यादृच्छिक चर  $x$  का माध्य और प्रसरण क्रमशः 4 और 2 है। तो  $P(x = 1)$  है ?

(5)

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- (a)  $\frac{1}{32}$       (b)  $\frac{1}{16}$       (c)  $\frac{1}{8}$       (d)  $\frac{1}{4}$

41. A unit vector perpendicular to both  $\hat{i} + \hat{j}$  and  $\hat{j} + \hat{k}$  is equal to

(सदिश  $\hat{i} + \hat{j}$  और  $\hat{j} + \hat{k}$  पर लम्ब, इकाई सदिश बराबर है ?)

- (a)  $\hat{i} - \hat{j} + \hat{k}$       (b)  $\hat{i} + \hat{j} + \hat{k}$       (c)  $\frac{\hat{i} + \hat{j} + \hat{k}}{3}$       (d)  $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

42.  $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx =$

- (a)  $x - \tan x + c$       (b)  $\tan x + x + c$       (c)  $\tan x - x + c$       (d)  $-\tan x - x + c$

43. A pair of dice are rolled. The Probability of obtaining an even prime number on each die is—  
(एक जोड़ा पासा फेंका जाता है। दोनों पर सम रुद्र संख्या आने की प्रायिकता है ?)

- (a)  $\frac{1}{36}$       (b)  $\frac{1}{12}$       (c)  $\frac{1}{6}$       (d) 0

44.  $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx =$

- (a)  $\frac{1}{\sin x + \cos x} + c$       (b)  $\log(\sin x + \cos x) + c$   
(c)  $\log |\sin x - \cos x| + c$       (d)  $\frac{1}{(\sin x + \cos x)^2} + c$

45. If (यदि)  $x = \frac{1-t^2}{1+t^2}$  and (और)  $y = \frac{2t}{1+t^2}$  then (तो)  $\frac{dy}{dx} =$

- (a)  $\frac{-y}{x}$       (b)  $\frac{y}{x}$       (c)  $\frac{-x}{y}$       (d)  $\frac{x}{y}$

46. The number of arbitrary constant in the general solution of a differential equation of fourth order is

कोटि 4 के अवकल समीकरण के व्यापक हल में स्वेच्छ अचर की संख्या है ।

- (a) 0      (b) 2      (c) 3      (d) 4

47. The general solution of differential equation  $\frac{dy}{dx} = e^{x+y}$  is—

(अवकल समीकरण  $\frac{dy}{dx} = e^{x+y}$  का व्यापक हल है ?)

- (a)  $e^x + e^y = c$       (b)  $e^x + e^{-y} = c$       (c)  $e^{-x} + e^y = c$       (d)  $e^{-x} + e^{-y} = c$

48.  $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx =$

- (a)  $\frac{\pi}{4}$       (b)  $-\frac{\pi}{4}$       (c) 0      (d)  $\frac{\pi}{2}$

(6)

49. If (यदि)  $P(A \cup B) = 0.8$  and (और)  $P(A \cap B) = 0.3$  then (तो)  $P(\bar{A}) + P(\bar{B}) =$   
 (a) 0.3                    (b) 0.5                    (c) 0.7                    (d) 0.9

50. Which of the following is the value  $c$  of Rolle's theorem when  $f(x) = 2x^3 - 5x^2 - 4x + 3, x \in \left[\frac{1}{3}, 3\right]$  is—  
 (निम्न में से कौन-सा राँले प्रमेय के नियतांक  $c$  का मान है जबकि  $f(x) = 2x^3 - 5x^2 - 4x + 3, x \in \left[\frac{1}{3}, 3\right]$ )  
 (a)  $-\frac{1}{3}$                     (b)  $\frac{2}{3}$                     (c) -2                    (d) 2

## **SECTION ( ਖਣਡ )-II**

## **Non-Objective Questions ( गैर वस्तुनिष्ट प्रश्न )**

**Time : [2 Hrs**

Full Marks : 50

समय : 2 घंटा

पूर्णिक : 50

Question number 1 to 22 are of short Answer type. Each question carry 2 marks. Answer any 15 Question.

[प्रश्न संख्या 1 से 22 तक लघुउत्तरीय प्रकार के हैं। प्रत्येक के लिए 2 अंक निर्धारित हैं। किन्हीं 15 प्रश्नों के उत्तर दें)  $15 \times 2 = 30$

$$1. \quad \text{Solve for } x : \tan^{-1} \left( \frac{1-x}{1+x} \right) = \frac{1}{2} \tan^{-1} x$$

$$(\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x \text{ तो } x \text{ का मान निकालें)$$

2. Find a matrix  $X$  such that  $2A + B + X = 0$ , where  $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$

(यदि  $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$  और  $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$  इस प्रकार है कि  $2A + B + X = 0$  तो आव्यूह  $X$  का मान ज्ञात करें।)

### **3. Using properties of determinant Prove that—**

( सारणिक के गुणों का प्रयोग कर सिद्ध करें । )

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

4. Check the continuity of the function  $f(x) = \begin{cases} \frac{|\sin x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$  at  $x = 0$

(फलन  $f(x) = \begin{cases} \frac{|\sin x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$  के सतता की जाँच  $x=0$  पर करें।)

(7)

5. Find the value of  $\lambda$  for which the planes  $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 13$  and  $\vec{r} \cdot (\lambda\hat{i} + 2\hat{j} - 7\hat{k}) = 9$  are perpendicular to each other.

$\lambda$  का मान ज्ञात करें जिससे तल  $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 13$  और  $\vec{r} \cdot (\lambda\hat{i} + 2\hat{j} - 7\hat{k}) = 9$  परस्पर लम्बवत हो।

6. Prove that (प्रमाणित कीजिए)

(a)  $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, x \in [-1, 1]$

(b)  $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, x \in R$

7. Find the point at which the tangent to the curve  $y = \sqrt{4x-3} - 1$  has its slope  $\frac{2}{3}$ .

(वक्र  $y = \sqrt{4x-3} - 1$  के किसी बिन्दु पर स्पर्श रेखा की ढाल  $\frac{2}{3}$  है, उस बिन्दु को ज्ञात कीजिए।)

8. Find the projection of the vector  $\hat{i} + \hat{j} + \hat{k}$  along the vector  $\hat{j}$

(सदिश  $\hat{i} + \hat{j} + \hat{k}$  का सदिश  $\hat{j}$  की दिशा में प्रक्षेप ज्ञात करें )

9. Show that the lines

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$  and  $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$  intersect each other. Find their point of intersection.

(दिखाइ कि  $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$  और  $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$  एक दूसरे को प्रतिच्छेद करती है। उसके प्रतिच्छेद बिन्दु को ज्ञात कीजिए।)

10. Find the equation of straight line parallel to  $(2\hat{i} - \hat{j} + 3\hat{k})$  and passing through the point  $(5, -2, 4)$ .

(( $2\hat{i} - \hat{j} + 3\hat{k}$ ) के समानान्तर और बिन्दु  $(5, -2, 4)$  से गुजरने वाली सरल रेखा का समीकरण ज्ञात करें।

11. If  $A$  and  $B$  are independent events, then Prove that  $A$  and  $B'$  are independent event.

(यदि  $A$  और  $B$  स्वतंत्र घटनाएँ हैं, तो सिद्ध करें कि  $A$  और  $B'$  स्वतंत्र घटना हैं।)

12. If two events  $A$  and  $B$  are such that  $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$  and  $P(A \cup B) = \frac{2}{3}$ . Show that  $A$  and  $B$  are mutually independent.

(यदि दो घटना  $A$  और  $B$  इस प्रकार हैं कि  $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$  और  $P(A \cup B) = \frac{2}{3}$ . दिखाइए कि  $A$  और  $B$  आपस में स्वतंत्र हैं।)

13. Evaluate (ज्ञात करें) :  $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

(8)

14. Evaluate (ज्ञात करें) :  $\int_1^4 f(x) dx$ , where (जहाँ)  $f(x) = \begin{cases} 2x+8, & 1 \leq x \leq 2 \\ 6x, & 2 \leq x \leq 4 \end{cases}$

15. Evaluate (ज्ञात करें) :  $\int_0^1 x(1-x)^{23} dx$

16. Find differential equation of the family of curves  $y = Ae^x + Be^{-x}$ , Where  $A$  and  $B$  are arbitrary constant.

(वक्र  $y = Ae^x + Be^{-x}$  के लिए अवकल समीकरण ज्ञात करें, जहाँ  $A$  and  $B$  नियतांक हैं।)

17. Solve the differential equation

$$\frac{dy}{dx} = 1 - x + y - xy$$

(अवकल समीकरण  $\frac{dy}{dx} = 1 - x + y - xy$  को हल करें)

18. Let  $N$  be the set of natural numbers and relation  $R$  on  $N$  be defined as

$R = \{(x, y) : x, y \in N \text{ and } x \text{ divides } y\}$  Examine whether  $R$  is Reflexive, symmetric & transitive.

(माना कि  $N$  प्राकृतिक संख्याओं का समुच्चय है और  $N$  पर  $R$  एक सम्बन्ध इस प्रकार परिभाषित है।  $R = \{(x, y) : x, y \in N \text{ और } x, y \text{ को विभाजित करता है}\}$ ,  $R$  के स्वतुल्य, सममित और संक्रमक होने की जाँच करें।)

19. Verify Roll's theorem for

$$f(x) = x^3(x-1)^2 \text{ in } [0, 1]$$

( $f(x) = x^3(x-1)^2$  के लिए  $[0, 1]$  में राँले प्रमेय के सत्यता की जाँच करें।)

20. If (यदि)  $x^y = y^x$ , find (ज्ञात करें)  $\frac{dy}{dx}$

21. If  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ , then show that  $\vec{a}$  is perpendicular on  $\vec{b}$ .

(यदि  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ , तो दिखाइए कि  $\vec{a}, \vec{b}$  पर लम्ब हैं।)

22. Let  $f: R \rightarrow R$  be a function defined by  $f(x) = \frac{x^2}{x^2 + 1}, \forall x \in R$ . Find  $f^{-1}$

(माना  $f: R \rightarrow R$  एक फलन  $f(x) = \frac{x^2}{x^2 + 1}, \forall x \in R$  द्वारा परिभाषित है।  $f^{-1}$  ज्ञात करें।)

प्रश्न संख्या 23 से 26 दीर्घउत्तरीय कोटि के हैं। प्रत्येक के लिए 5 अंक निर्धारित हैं। प्रत्येक प्रश्न या उसके विकल्प में से किसी एक का उत्तर दें।

Question number 23 to 26 are of long answer type. Each question carry 5 marks. Answer every question or alternative of it. [4 × 5]

23. Find the equation of the curve passing through the point  $(1, -1)$  whose differential equation is

(9)

$$xy \frac{dy}{dx} = (x+2)(y+2)$$

बिन्दु (1, -1) से गुजरने वाली बक्र का समीकरण ज्ञात करें। जिसका अवकल समीकरण  
 $xy \frac{dy}{dx} = (x+2)(y+2)$  है।

Or, (या)

Find the area of the region included between the parabola  $y^2 = 4ax$  and  $x^2 = 4ay$ . Where  $a > 0$   
(परवलय  $y^2 = 4ax$  और  $x^2 = 4ay$ . से धिरे क्षेत्र का क्षेत्रफल ज्ञात करें, जहाँ  $a > 0$ )

24. Evaluate (ज्ञात करें) :  $\int \frac{x}{(x^2+1)(x+1)} dx$

Or, (या)

$$\int_0^{\pi/4} \log(1+\tan x) dx$$

25. Find the vector equation of the plane through the points  $A(3, 5, -1)$ ;  $B(-1, 5, 7)$  and parallel to  
the vector  $3\hat{i} - \hat{j} + 7\hat{k}$ .

(बिन्दु  $A(3, -5, -1)$ ;  $B(-1, 5, 7)$  से गुजरने वाली और सदिश  $3\hat{i} - \hat{j} + 7\hat{k}$  के समानान्तर तल का  
सदिश समीकरण ज्ञात करें।

Or, (या)

Prove that (सिद्ध करें)

$$[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$$

26. Solve the following LPP graphically, Maximize  $z = 10x + 6y$   
subjected to

$$3x + y \leq 12$$

$$2x + 5y \leq 34, x \geq 0, y \geq 0$$

निम्न LPP को ग्राफीय विधि से हल करें।

$$\text{अधिकतमीकरण } z = 10x + 6y$$

$$\text{जबकि } 3x + y \leq 12$$

$$2x + 5y \leq 34, x \geq 0, y \geq 0$$

Or, (या)

$$\text{Minimize (न्यूनतमीकरण)} = 20x + 10y$$

$$\text{subjected to } x + 2y \leq 40$$

$$\text{जबकि } 3x + y \geq 30$$

$$4x + 3y \geq 60, x \geq 0, y \geq 0$$

(10)

## SOLUTION Answer Key

### Objective Question :

1. (d)	2. (a)	3. (b)	4. (a)	5. (b)	6. (a)	7. (a)	8. (c)	9. (d)	10. (a)
11. (a)	12. (a)	13. (a)	14. (c)	15. (d)	16. (b)	17. (d)	18. (a)	19. (b)	20. (b)
21. (a)	22. (b)	23. (b)	24. (a)	25. (b)	26. (b)	27. (c)	28. (a)	29. (b)	30. (c)
31. (a)	32. (b)	33. (c)	34. (a)	35. (b)	36. (b)	37. (c)	38. (c)	39. (a)	40. (b)
41. (c)	42. (c)	43. (a)	44. (b)	45. (c)	46. (d)	47. (b)	48. (a)	49. (a)	50. (d)

### Solution to Short Answer type question :

1. Given,  $\tan^{-1} \frac{1-x}{1+x} = \frac{1}{2} \tan^{-1} x$

$$\begin{aligned}\Rightarrow \tan^{-1} x &= 2 \tan^{-1} \frac{1-x}{1+x} = \tan^{-1} \frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2} \\ &= \tan^{-1} \frac{2(1-x)(1+x)}{(1+x)^2 - (1-x)^2} \quad \left[ \because 2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2} \right] \\ &= \tan^{-1} 2(1-x^2)\end{aligned}$$

$$\therefore \tan^{-1} x = \tan^{-1} \frac{(1-x^2)}{2x}$$

$$\therefore x = \frac{1-x^2}{2x}$$

$$\Rightarrow 2x^2 = 1 - x^2 \Rightarrow 3x^2 = 1$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{3}} \quad \therefore x = \frac{1}{\sqrt{3}} \quad (\because x > 0)$$

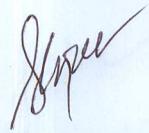
2. We have

$$2A + B + X = 0 \Rightarrow X = -(2A + B)$$

$$\begin{aligned}\text{Now, } 2A + B &= 2 \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} \\ &= \begin{bmatrix} -2+3 & 4+(-2) \\ 6+1 & 8+5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 7 & 13 \end{bmatrix}\end{aligned}$$

$$\therefore X = -(2A + B) = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$$

(11)



$$\begin{aligned}
 3. \text{ Let } \Delta &= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} \\
 &= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-b \\ bc & ca-bc & ab-ca \end{vmatrix} [\text{By } C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_2] \\
 &= \begin{vmatrix} 1 & 0 & 0 \\ a & -(a-b) & -(b-c) \\ bc & c(a-b) & a(b-c) \end{vmatrix} = (a-b)(b-c) \begin{vmatrix} 1 & 0 & 0 \\ a & -1 & -1 \\ bc & c & a \end{vmatrix}
 \end{aligned}$$

[Taking common  $(a-b)$  from  $C_2$  &  $(b-c)$  from  $C_3$ ]  
 $= (a-b)(b-c) \cdot 1 (-a+c)$   
 $= (a-b)(b-c)(c-a) \text{ Proved.}$

4. We have  $f(0) = 1$

$$\begin{aligned}
 \text{RHL} &= \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{|\sin(0+h)|}{0+h} = \lim_{h \rightarrow 0} \frac{|\sin h|}{h} = \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \\
 \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{|\sin(-h)|}{-h} = \lim_{h \rightarrow 0} \frac{|-\sin h|}{-h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin h}{h} = -1
 \end{aligned}$$

$\therefore \lim_{x \rightarrow 0^+} f(x) \neq \lim_{x \rightarrow 0^-} f(x)$ . So,  $\lim_{x \rightarrow 0} f(x)$  does not exist.

Hence,  $f(x)$  is discontinuous at  $x = 0$

5. We know that two plane  $\vec{r} \cdot \vec{n}_1 = q_1$  and  $\vec{r} \cdot \vec{n}_2 = q_2$  are perpendicular to each other if and only if  $\vec{n}_1 \cdot \vec{n}_2 = 0$

Here,  $\vec{n}_1 = (\hat{i} + 2\hat{j} + 3\hat{k})$  and  $\vec{n}_2 = (\lambda\hat{i} + 2\hat{j} - 7\hat{k})$

Given that planes are perpendicular to each other

$$\text{So, } \vec{n}_1 \cdot \vec{n}_2 = 0$$

$$\Rightarrow (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (\lambda\hat{i} + 2\hat{j} - 7\hat{k}) = 0$$

$$\Rightarrow 1 \times \lambda + 2 \times 2 + 3 \times -7 = 0$$

$$\Rightarrow \lambda = 17$$

Hence, required value of  $\lambda$  is 17.

6. (a) Let  $\sin^{-1} x = \theta$  ... (i)  
 $\Rightarrow x = \sin \theta$

$$\Rightarrow x = \cos\left(\frac{\pi}{2} - \theta\right) \quad [:-1 \leq x \leq 1 \text{ and } \frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2} \Rightarrow \frac{\pi}{2} - \theta \in [0, \pi]]$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{2} - \theta \quad [x \in [-1, 1] \text{ and } \left(\frac{\pi}{2} - \theta\right) \in [0, \pi]]$$

(12)

*Ans*

$$\Rightarrow \theta + \cos^{-1}x = \frac{\pi}{2} \quad \dots \text{(ii)}$$

By (i) & (ii)

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

$$(ii) \text{ Let } \tan^{-1}x = \theta \quad \dots \text{(iii)}$$

$$\text{then, } x \in R \text{ and } -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} - \theta \in (0, \pi)$$

$$\text{So, } x = \tan \theta$$

$$\Rightarrow x = \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\Rightarrow \cot^{-1}x = \frac{\pi}{2} - \theta \quad [:\frac{\pi}{2} - \theta \in (0, \pi)]$$

$$\Rightarrow \theta + \cot^{-1}x = \frac{\pi}{2} \quad \dots \text{(2)}$$

By (1) & (2)

$$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$

$$7. \text{ Given curve is } y = \sqrt{4x-3} - 1$$

Differentiating, we have

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4x-3}} \times 4 = \frac{2}{\sqrt{4x-3}}$$

Also, slope of tangent at any point on the curve is  $\frac{2}{3}$

$$\therefore \frac{dy}{dx} = \frac{2}{3}$$

$$\Rightarrow \frac{2}{\sqrt{4x-3}} = \frac{2}{3} \Rightarrow \sqrt{4x-3} = 3$$

$$\Rightarrow 4x - 3 = 9$$

$$\Rightarrow 4x = 12 \therefore x = 3$$

$$\text{So, } y = \sqrt{4 \times 3 - 3} - 1 = \sqrt{9} - 1 = 2$$

So, the require point is (3, 2)

$$8. \text{ The projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

(13)

*[Signature]*

Here  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = \vec{j}$

So,  $\vec{a} \cdot \vec{b} = 1$  and  $|\vec{b}| = 1$

So, Projection of  $\hat{i} + \hat{j} + \hat{k}$  along  $\vec{J} = \frac{1}{1} = 1$  unit

9. The given lines will intersect if for some particular values of  $\lambda$  and  $\mu$ , the values of  $\vec{r}$  (given) of both lines are same.

$$\text{So, } (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j}) = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$

$$\Rightarrow (1+3\lambda)\hat{i} + (1-\lambda)\hat{j} - \hat{k} = (4+2\mu)\hat{i} + (3\mu-1)\hat{k}$$

$$\Rightarrow 1 + 3\lambda = 4 + 2\mu, 1 - \lambda = 0 \text{ and } 3\mu - 1 = -1$$

$$\Rightarrow 3\lambda - 2\mu = 3$$

$$\lambda = 1$$

$$\text{and } \mu = 0$$

... (i)

... (ii)

... (iii)

Clearly,  $\lambda = 1$  and  $\mu = 0$  also satisfy (1)

Putting  $\lambda = 1$  in  $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(3\hat{i} - \hat{j})$ , we get

$$\vec{r} = (4\hat{i} + 0\hat{j} - \hat{k})$$

Hence, point of intersection of the given lines is  $(4, 0, -1)$

10. Let co-ordinate of any point  $p$  be  $(5, -2, 4)$

$$\text{then } \vec{OP} = 5\hat{i} - 2\hat{j} + 4\hat{k} = \vec{a} \quad [\text{Let O being origin}]$$

$$\text{and Let } \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$$

Hence, Equation of line passing through  $\vec{a}$  and parallel to  $\vec{b}$  is  $\vec{r} = \vec{a} + t \cdot \vec{b}$

$$\text{So, } x\hat{i} + y\hat{j} + z\hat{k} = (5\hat{i} - 2\hat{j} + 4\hat{k}) + t(2\hat{i} - \hat{j} + 3\hat{k})$$

$$[\text{in cartesian form } \frac{x-5}{2} = \frac{y+2}{-2} = \frac{z-4}{3}]$$

11. Given,  $A$  and  $B$  are independent event

$$\text{So, } P(A \cap B) = P(A), P(B)$$

Now, we can write

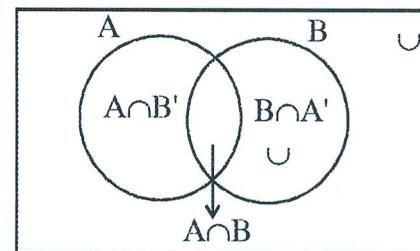
$$A = (A \cap B) \cup (A \cap B')$$

$$\therefore P(A) = P(A \cap B) + P(A \cap B') \\ = P(A) \cdot P(B) + P(A) \cdot P(B')$$

$$\Rightarrow P(A \cap B') = P(A) - P(A) \cdot P(B) = P(A) [1 - P(B)] \\ = P(A) \cdot P(B')$$

So,  $A$  and  $B'$  are independent event.

12. For set  $A$  &  $B$ , Addition theorem is given by



... (1)

(14)

*Spur*

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\Rightarrow \frac{2}{3} = \frac{1}{2} + \frac{1}{3} - P(A \cup B)$$

$$\therefore P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{2}{3} = \frac{1}{6}$$

$$\text{Now, } P(A \cap B) = \frac{1}{6} = \frac{1}{2} \cdot \frac{1}{3} = P(A) \cdot P(B)$$

So, A & B are mutually independent.

$$13. \text{ Let } I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots(1)$$

$$\text{Then, } I = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)} + \sqrt{\cos\left(\frac{\pi}{2}-x\right)}} dx = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \dots(2)$$

(1) + (2) gives

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \int_0^{\pi/2} dx$$

$$\Rightarrow 2I = \frac{\pi}{2} \quad \therefore I = \frac{\pi}{4}$$

$$14. \int_1^4 f(x) dx = \int_1^2 (2x+8) dx + \int_2^4 6x dx$$

$$= [2x^2 + 8x]_1^2 + [3x^2]_2^4$$

$$= [4 + 16 - (1 + 8)] + [3 \times 16 - 3 \times 4]$$

$$= 11 + 36 = 47$$

$$15. \text{ Let } z = 1-x, \text{ then } dz = -dx \text{ and } x = 1-z$$

when  $x = 0, z = 1$  and when  $x = 1, z = 0$

$$\therefore \int_0^1 x(1-x)^{23} dx = - \int_1^0 (1-z)z^{23} dz = - \int_1^0 z^{23} - z^{24} dx$$

$$= - \left[ \frac{z^{24}}{24} - \frac{z^{25}}{25} \right]_1^0 = - \left[ 0 - \left( \frac{1}{24} - \frac{1}{25} \right) \right] = \frac{1}{600}$$

$$16. \text{ Given Curve is } y = A \cdot e^x + B \cdot e^{-x} \quad \dots(1)$$

Differentiating both sides w.r.t. 'x', we have

$$\frac{dy}{dx} = Ae^x - Ae^{-x} \quad \dots(2)$$

Again differentiating, w.r.t. 'x'

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*Shyamal*

$$\frac{d^2y}{dx^2} = Ae^x + Be^{-x}$$

$$\text{i.e., } \frac{d^2y}{dx^2} = y$$

So,  $\frac{d^2y}{dx^2} - y = 0$  which is required differential Equation.

17. Given differential equation is

$$\frac{dy}{dx} = 1 - x + y - xy = (1 - x) + y(1 - x)$$

$$\text{Or, } \frac{dy}{dx} = (1 - x)(1 + y)$$

$$\text{Or, } \frac{dy}{1+y} = (1-x) dx \text{ [By separation of variable.]}$$

on Integration,

$$\int \frac{dy}{1+y} = \int (1-x) dx$$

$$\text{Or, } \log |1+y| = x - \frac{x^2}{2} + k$$

18. (i)  $x$  divides  $x$  i.e.  $xRx, \forall x \in N$

$\Rightarrow R$  is Reflexive

(ii) 2 divides 6 i.e.  $2R6$  i.e.  $(2, 6) \in R$  but  $(6, 2) \notin R$  as 6 does not divide 2.

∴  $R$  is not symmetric

(iii) Let  $x$  divides  $y$  and  $y$  divides  $z$

i.e.  $xRy$  and  $yRz$

$\Rightarrow k_1 x = y$  &  $k_2 y = z$  ( $k_1, k_2$  are positive integer)

$\Rightarrow k_1 k_2 x = k_1 y = z$

$\Rightarrow x$  divides  $z \Rightarrow xRz$

So,  $R$  is transitive.

19. Given,  $f(x) = x^3(x-1)^2$

$$f'(x) = 3x^2(x-1)^2 + x^3 \cdot 2(x-1) = x^2(x-1)(3(x-1) + 2x)$$

$$\text{Or, } f'(x) = x^2(x-1)(5x-3)$$

Clearly,  $f(x)$  is differentiable and continuous for all  $n$ .

So, (ii)  $f(x)$  is differentiable in  $(0, 1)$  also.

$$(iii) f(0) = 0 \text{ and } f(1) = 0 \therefore f(0) = f(1)$$

Hence, all the condition of Rolle's theorem are satisfied for  $f(x)$  in  $[0, 1]$ .

$$\text{So, } f'(c) = 0$$

$$\Rightarrow c^2(c-1)(5c-3) = 0 \Leftrightarrow c = 0, 1, \frac{3}{5}$$

$$\text{But } 0 < c < 1 \therefore c = \frac{3}{5}$$

(16)

Thus, there exist at least one  $c$ ,  $c = \frac{3}{5}$  between 0 & 1 such that  $f'(c) = 0$

Hence, Rolle's theorem has been verified.

20. Given,  $x^y = y^x$

Taking logarithm on both sides, we get

$$y \log x = x \log y$$

Differentiating both sides, w.r.t.  $x$

$$y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1$$

$$\Rightarrow \frac{y}{x} + (\log x) \cdot \frac{dy}{dx} = \frac{x}{y} \cdot \frac{dy}{dx} + \log y$$

$$\Rightarrow \frac{dy}{dx} \left[ \log x - \frac{x}{y} \right] = \log y - \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} \left[ \frac{y \log x - x}{y} \right] = \frac{x \log y - y}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y(x \log y - y)}{x(y \log x - x)}$$

21. Given that,  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

squaring both side,

$$(\vec{a} + \vec{b})^2 = (\vec{a} - \vec{b})^2$$

$$\Rightarrow a^2 + b^2 + 2\vec{a} \cdot \vec{b} = a^2 + b^2 - 2\vec{a} \cdot \vec{b}$$

$$\Rightarrow 4\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \cdot \vec{b} = 0$$

Thus,  $\vec{a}$  is perpendicular to  $\vec{b}$ .

22. Let  $f: R \rightarrow R$  defined by  $f(x) = \frac{x^2}{x^2 + 1}$

Taking  $1, -1 \in R$

$$\text{Clearly, } f(1) = f(-1) = \frac{1}{2}$$

Hence,  $f$  is a many one function.

Therefore  $f^{-1}$  does not exist

**Long Answer Type Question (दीर्घ उत्तरीय प्रश्न)**

23. Given differential equation is

$$xy \frac{dy}{dx} = (x+2)(y+2)$$

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$$\therefore \frac{y}{y+2} dy = \frac{x+2}{x} dx, y \neq -2, x \neq 0$$

$$\text{or, } \left(1 - \frac{2}{y+2}\right) dy = \left(1 + \frac{2}{x}\right) dx$$

Integrating,

$$\int \left(1 - \frac{2}{y+2}\right) dy = \int \left(1 + \frac{2}{x}\right) dx$$

$$\text{Or, } y - 2\log|y+2| = x + 2\log|x| + k \quad \dots(1)$$

Since, curve passes through  $(1, -1)$

So, by (1)

$$-1 - 2\log 1 = 1 + 2\log 1 + k$$

$$\therefore k = -2$$

Hence, required equation of the curve is

$$y - 2\log|y+2| = x + 2\log|x| - 2$$

Or,

$$\text{Given Parabola are } y^2 = 4ax$$

$$\text{and } x^2 = 4ay \quad \dots(2)$$

Solving (1) & (2) to get point of intersection

So, Putting  $x = \frac{y^2}{4a}$  from (1) in (2), we get

$$\frac{y^4}{16a^2} = 4ay \Rightarrow y^4 - 64a^3y = 0$$

$$\Rightarrow y(y^3 - 64a^3) = 0$$

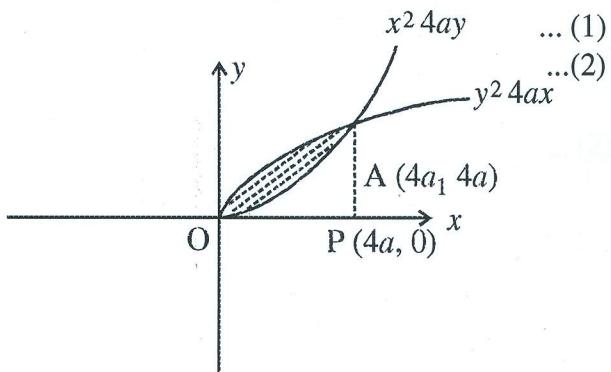
$$\Rightarrow y = 0 \text{ or, } y = 4a$$

When  $y = 0, x = 0$  and when  $y = 4a, x = 4a$

So, Required Area OBAO

$$\begin{aligned} &= \int_0^{4a} (y_1 - y_2) dx \\ &= \int_0^{4a} y dx \quad (\text{for } y^2 = 4ax) - \int_0^{4a} y dx \quad (\text{for } x^2 = 4ay) \\ &= \int_0^{4a} 2\sqrt{ax} dx - \int_0^{4a} \frac{x^2}{4a} dx \\ &= \left[ 2\sqrt{a} \cdot \frac{2}{3} \cdot x^{3/2} \right]_0^{4a} - \frac{1}{4a} \left[ \frac{x^3}{3} \right]_0^{4a} \\ &= \left[ \frac{4\sqrt{a}}{3} (4a)^{3/2} - \frac{1}{12a} \times 64a^3 \right] \end{aligned}$$

(18)



$$= \left( \frac{32a^2}{3} - \frac{16a^2}{3} \right) = \frac{16a^2}{3} \text{ sq. unit.}$$

24. Let  $I = \int \frac{x}{(x^2+1)(x+1)} dx$

Now,  $\frac{x}{(x^2+1)(x+1)} = \frac{1}{2} \left[ \frac{x+1}{x^2+1} - \frac{1}{x+1} \right]$  (Partial fraction)

So,  $I = \frac{1}{2} \int \frac{x}{x^2+1} dx + \int \frac{dx}{x^2+1} - \int \frac{dx}{x+1}$

$$= \frac{1}{4} \log(x^2+1) + \frac{1}{2} \tan^{-1} x - \frac{1}{2} \log|x+1| + k$$

Or,

Let  $f(x) = \log(1 + \tan x)$  ... (1)

Now,  $f\left(\frac{\pi}{4} - x\right) = \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right] = \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right)$

So,  $f\left(\frac{\pi}{4} - x\right) = \log\frac{2}{1 + \tan x}$  ... (2)

By, (1) + (2), we have

$$\begin{aligned} f(x) + f\left(\frac{\pi}{4} - x\right) &= \log(1 + \tan x) + \log\left(\frac{2}{1 + \tan x}\right) \\ &= \log 2 \end{aligned}$$

Let  $I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$

So,  $I = \int_0^{\frac{\pi}{4}} \log\frac{2}{1 + \tan x} dx$

Adding,

$$2I = \int_0^{\frac{\pi}{4}} \left[ f(x) + f\left(\frac{\pi}{4} - x\right) \right] dx$$

$$= \int_0^{\frac{\pi}{4}} \log 2 \cdot dx = \frac{\pi}{4} \log 2$$

$$\therefore I = \frac{\pi}{8} \log 2$$

(19)

*Singh*

25. The position vector of given points  $A$  and  $B$  are respectively  $(3\hat{i} - \hat{j} + 7\hat{k})$  and  $(-\hat{i} + 5\hat{j} + 7\hat{k})$

Let  $P$  be any point on the plane with position vector  $x\hat{i} + 5\hat{j} + 2\hat{k}$

$$\text{So, } \vec{AP} = (x-3)\hat{i} + (y+5)\hat{j} + (z+1)\hat{k}$$

$$\text{and } \vec{AB} = -4\hat{i} + 10\hat{j} + 8\hat{k}.$$

Clearly,  $\vec{AP}$ ,  $\vec{AB}$  and given vector  $3\hat{i} - \hat{j} + 7\hat{k}$  are co-planar.

So, Equation of the require plane is

$$\begin{vmatrix} x-3 & y+5 & z+1 \\ -4 & 10 & 8 \\ 3 & -1 & 7 \end{vmatrix} = 0$$

$$\text{Or, } 78(x-3) + 52(y+5) - 26(z+1) = 0$$

$$\text{Or, } 78x + 52y - 26z = 0$$

$$\text{Or, } 3x + 2y - z = 0$$

The vector rquation of require plane is

$$\vec{r} \cdot (3\hat{i} + 2\hat{j} - \hat{k}) = 0$$

Or,

$$\begin{aligned} \text{LHS } & [\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] \\ &= (\vec{a} \times \vec{b}) \cdot \{(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})\} \\ &= (\vec{a} \times \vec{b}) \cdot \{\vec{d} \times (\vec{c} \times \vec{a})\} \text{ where } \vec{d} = \vec{b} \times \vec{c} \\ &= (\vec{a} \times \vec{b}) \cdot \{(\vec{d} \cdot \vec{a}) \vec{c} - (\vec{d} \cdot \vec{c}) \vec{a}\} \\ &= (\vec{a} \times \vec{b}) \cdot [(\vec{b} \times \vec{c}) \cdot \vec{a} \vec{c} - (\vec{b} \times \vec{c}) \cdot \vec{c} \vec{a}] \\ &= (\vec{a} \times \vec{b}) \cdot [[\vec{a} \vec{b} \vec{c}] \vec{c} - [\vec{c} \vec{b} \vec{c}] \vec{a}] \\ &= (\vec{a} \times \vec{b}) \cdot [[\vec{a} \vec{b} \vec{c}] \vec{c} - 0] \\ &= [\vec{a} \vec{b} \vec{c}] \{(\vec{a} \times \vec{b}) \cdot \vec{c}\} = [\vec{a} \vec{b} \vec{c}] \{\vec{a} \cdot (\vec{b} \times \vec{c})\} \\ &= [\vec{a} \vec{b} \vec{c}] \cdot [\vec{a} \vec{b} \vec{c}] = [\vec{a} \vec{b} \vec{c}]^2 = \text{RHS} \end{aligned}$$

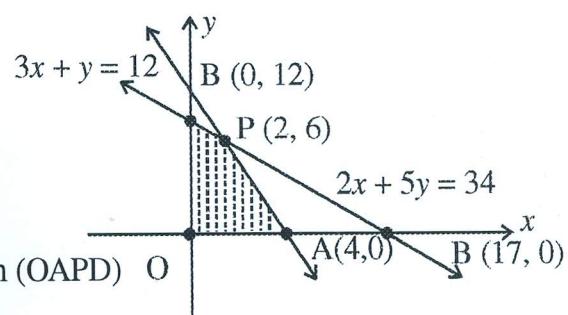
26. Changing the constraints into equation

$$3x + y = 12 \text{ and } 2x + 5y = 34$$

We draw the line of above equation which is as—

The feasible region (shaded region) is the bounded region (OAPD)

The vertices of the feasible region are



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$0(0, 0)$ ,  $A(4, 0)$ ,  $P(2, 6)$  and  $D\left(0, \frac{34}{5}\right)$

Given function :  $z = 10x + 6y$

At  $0(0, 0)$ ,  $z = 0$

At  $A(4, 0)$ ,  $z = 40$

At  $P(2, 6)$ ,  $z = 56$

At  $D\left(0, \frac{34}{5}\right)$ ,  $z = \frac{204}{5} = 40.8$

Hence,  $Z_{\max.} = 56$  at  $x = 2, y = 6$

Or,

First we draw the lines

$$x + 2y = 40, 3x + y = 30, 4x + 3y = 60$$

The feasible region (Shaded region) is the bounded region "EAQPA"

The vertices of the feasible region are

$E(15, 0), A(40, 0), Q(4, 18)$  and  $P(6, 12)$

Given objective function is

$$z = 20x + 10y$$

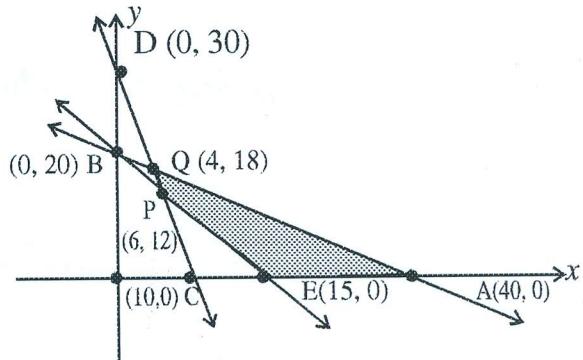
At,  $E(15, 0)$ ,  $z = 300$

At,  $A(40, 0)$ ,  $z = 800$

At,  $Q(4, 18)$ ,  $z = 260$

At,  $P(6, 12)$ ,  $z = 240$

Hence,  $Z_{\min.} = 240$  at  $x = 6, y = 12$



(21)

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