

## SET ( प्रारूप )-4

## SECTION ( खण्ड )-I

### Objective Questions ( वस्तुनिष्ट प्रश्न )

**Time : [1 Hrs**

Full Marks : 50

समय : 1 घंटा

पूर्णक : 50

There are 50 objective (one correct answer) question carrying one mark each. Choose the correct answer from the given option.

[इस खंड में 50 वस्तुनिष्ठ प्रश्न है। प्रत्येक प्रश्न के लिए 1 अंक निर्धारित है। दिए गये विकल्पों में से सही उत्तर चुनें। [50 × 1]



(1)

- (c)  $\frac{x}{\alpha} = \frac{y}{\beta} = \frac{z}{\gamma}$  (d) None of these (इनमें से कोई नहीं)
8. If  $l, m, n$  be the d.c. of any line then which of the following is true.  
 (यदि  $l, m, n$  किसी रेखा का दिक् कोज्या है तो निम्न में कौन सा सत्य है ?)  
 (a)  $l + m + n = 1$  (b)  $l^2 + m^2 + n^2 = 1$  (c)  $\sqrt{l^2 + m^2 + n^2} = 0$  (d) None of these (इनमें से कोई नहीं)
9. If A & B are square matrix then  $(AB)' =$   
 (यदि A और B वर्ग आव्यूह हैं तो  $(AB)' =$ )  
 (a)  $A'B'$  (b)  $AB'$  (c)  $B'A'$  (d)  $A'B$
10. The value of  $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$  is equal to  
 ( $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$  का मान बराबर है ?)  
 (a)  $\frac{7\pi}{6}$  (b)  $\frac{5\pi}{6}$  (c)  $\frac{\pi}{3}$  (d)  $\frac{\pi}{6}$
11. If (यदि)  $y = \log \{\log (\log x)\}$  then (तो)  $\frac{dy}{dx} =$   
 (a)  $\frac{1}{\log(\log x)}$  (b)  $\frac{1}{x \log x \cdot \log(\log x)}$   
 (c)  $\frac{1}{x \log(\log x)}$  (d) None of these (इनमें से कोई नहीं)
12. The direction cosine of line joining  $(1, -1, 1)$  and  $(-1, 1, 1)$  are which of the following ?  
 बिन्दु  $(1, -1, 1)$  और  $(-1, 1, 1)$  को मिलानेवाली रेखा का दिक् कोज्या निम्न में से कौन है ?  
 (a)  $(2, -2, 0)$  (b)  $(1, -1, 0)$  (c)  $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$  (d) None of these (इनमें से कोई नहीं)
13. If (यदि)  $y = a^x$  then (तो)  $\frac{d^2y}{dx^2} =$   
 (a)  $a^x \log a$  (b)  $a^x (\log a)^2$  (c)  $(a^x)^2 \log a$  (d) None of these (इनमें से कोई नहीं)
14. Which of the following is correct for the function  $f(x) = 2 + 4x^2 + 6x^4 + 8x^6$ .  
 (फलन  $f(x) = 2 + 4x^2 + 6x^4 + 8x^6$  के लिए निम्न में से कौन सत्य है ?)  
 (a) only one maximum value (केवल एक अधिकतम मान)  
 (b) Only one minimum (केवल एक न्यूनतम मान)  
 (c) No Maxima & minima (न अधिकतम न न्यूनतम) (d) None of these (इनमें से कोई नहीं)
15. 
$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} =$$
  
 (a) 0 (b)  $(x-y)(y-z)(z-x)$   
 (c)  $(y-x)(y-z)(z-x)$  (d) None of these (इनमें से कोई नहीं)
- (2)

16. If  $\vec{a} \times \vec{b} = 0$  and  $\vec{a} \cdot \vec{b} = 0$  then which of the following is true ?

(यदि  $\vec{a} \times \vec{b} = 0$  और  $\vec{a} \cdot \vec{b} = 0$  तो निम्न में से कौन सत्य है ?)

- (a)  $\vec{a} \perp \vec{b}$       (b)  $\vec{a} \parallel \vec{b}$       (c)  $\vec{a} = 0$  and  $\vec{b} = 0$       (d)  $\vec{a} = 0$  or  $\vec{b} = 0$

17.  $\int_1^{\sqrt{3}} \frac{1}{1+x^2} dx =$

- (a)  $\frac{\pi}{3}$       (b)  $\frac{\pi}{4}$       (c)  $\frac{\pi}{6}$       (d)  $\frac{\pi}{12}$

18. The projection of  $\hat{i} + 3\hat{j} + \hat{k}$  on  $2\hat{i} - 3\hat{j} + 6\hat{k}$  is equal to

( $2\hat{i} - 3\hat{j} + 6\hat{k}$  पर  $\hat{i} + 3\hat{j} + \hat{k}$  का प्रक्षेप बराबर है ?)

- (a)  $\frac{1}{7}$       (b)  $-\frac{1}{7}$       (c) 7      (d) -7

19.  $(\vec{a} \times \vec{b})^2 =$

- (a)  $a^2 + b^2 - (\vec{a} \cdot \vec{b})$       (b)  $a^2 b^2 - (\vec{a} \cdot \vec{b})^2$   
(c)  $a^2 b^2 - 2\vec{a} \cdot \vec{b}$       (d)  $\vec{a}^2 \cdot \vec{b}^2 + 2\vec{a} \cdot \vec{b}$

20. If  $\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$  then which of the following is true ?

(यदि  $\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$  तो निम्न में कौन सा सत्य है ?)

- (a)  $f(2a-x) = -f(x)$       (b)  $f(2a-x) = f(x)$   
(c)  $f(x)$  is an odd function ( $f(x)$  एक विषम फलन है)  
(d)  $f(x)$  is an even function ( $f(x)$  एक सम फलन है)

21. If  $\vec{a} = 2\hat{i} + \hat{j} - 8\hat{k}$  and  $\vec{b} = \hat{i} + 3\hat{j} - 4\hat{k}$  then magnitude of  $\vec{a} + \vec{b}$  is equal to

(यदि  $\vec{a} = 2\hat{i} + \hat{j} - 8\hat{k}$  और  $\vec{b} = \hat{i} + 3\hat{j} - 4\hat{k}$  तो  $\vec{a} + \vec{b}$  का परिमाप बराबर है ?)

- (a) 13      (b)  $\frac{13}{3}$       (c)  $\frac{3}{13}$       (d)  $\frac{6}{13}$

22.  $\int \frac{1}{x^2 - a^2} dx =$

- (a)  $\frac{1}{2a} \log \left| \frac{x-a}{x+a} \right|$       (b)  $\frac{1}{2a} \log \left| \frac{x+a}{x-a} \right|$       (c)  $\log(x + \sqrt{x^2 - a^2})$       (d)  $\log(x + \sqrt{x^2 + a^2})$

23. Equation of  $xy$ -plane is—

(3)

( $xy$ -तल का समीकरण है ?)

- (a) ( $x = 0$ )      (b) ( $y = 0$ )      (c) ( $z = 0$ )      (d) ( $xz = 0$ )

24. The line  $y = x + 1$  is tangent to the curve  $y^2 = 4x$  at point.

(बिन्दु जहाँ पर सरल रेखा  $y = x + 1$ , वक्र  $y^2 = 4x$  पर स्पर्श रेखा है ?)

- (a) (2, 1)      (b) (1, 2)      (c) (-1, 2)      (d) (1, -2)

25. The principal value of  $\text{cose}^{-1}(-2)$  is

( $\text{cose}^{-1}(-2)$  का मुख्य मान है ?)

- (a)  $-\frac{2\pi}{3}$       (b)  $\frac{\pi}{6}$       (c)  $\frac{2\pi}{3}$       (d)  $-\frac{\pi}{6}$

26.  $\int \log e^z dz =$

- (a)  $z \log z + x + k$       (b)  $z \log z - z + k$       (c)  $\log z + z + k$       (d)  $\log z - z + k$

27. The differential equation of family of lines passing through the origin is—

मूल बिन्दु से गुजरनेवाली रेखाओं के परिवार का अवकल समीकरण है ?

- (a)  $x \frac{dy}{dx} = y$       (b)  $y \frac{dy}{dx} = x$       (c)  $\frac{dy}{dx} = y$       (d)  $\frac{dy}{dx} = x$

28.  $\cos^{-1} \frac{1}{2} + 2 \sin^{-1} \frac{1}{2} =$

- (a)  $\frac{2\pi}{3}$       (b)  $\frac{\pi}{2}$       (c)  $\frac{4\pi}{3}$       (d) None of these (इनमें से कोई नहीं)

29. Domain of the function  $f(x) = \sqrt{\sin^{-1} x}$

(फलन  $\sqrt{\sin^{-1} x}$  का प्रांत है ?)

- (a) [0, 1]      (b) [-1, 1]      (c) [-1, 0]      (d) [0, 1]

30. Let  $A = \{1, 2, 3\}$  and a relation  $R$  on  $A$  defined by  $R = \{(2, 2), (3, 3), (2, 3), (3, 2), (3, 1), (2, 1)\}$  then what type of relation is  $R$ ?

(माना कि  $A = \{1, 2, 3\}$  और  $A$  पर एक सम्बन्ध  $R$  इस प्रकार परिभाषित है  $R = \{(2, 2), (3, 3), (2, 3), (3, 2), (3, 1), (2, 1)\}$  तो  $R$  किस प्रकार का सम्बन्ध है ?)

- (a) Reflexive (स्वतुल्य)      (b) Symmetric (सममित)  
(c) Equivalence (तुल्यता)      (d) Transitive (संक्रमक)

31.  $f: A \rightarrow B$  will be an into function if—

( $f: A \rightarrow B$  एक अनाच्छादक फलन है यदि—)

- (a)  $f(A) \subset B$       (b)  $f(A) = B$       (c)  $f(B) \subset A$       (d) None of these (इनमें से कोई नहीं)

32. If  $A = \{1, 2, 3\}$  then how many equivalence relations can be defined on  $A$  containing (1, 2) ?

(यदि  $A = \{1, 2, 3\}$  तो समुच्चय  $A$  पर {1, 2} को समाहित करने वाला कितने तुल्यता सम्बन्ध परिभाषित हो सकते है ?)

- (a) 8      (b) 10      (c) 16      (d) 20

33. If  $A = \{a, b, c\}$ ,  $B = \{1, 2, 3\}$  and  $f = \{(a, 1), (b, 2), (c, 2)\}$  then what type of function is  $f$ ?

(यदि  $A = \{a, b, c\}$ ,  $B = \{1, 2, 3\}$  और  $f = \{(a, 1), (b, 2), (c, 2)\}$  तो  $f$  किस प्रकार का फलन है ?)

- (a) one-one onto (एकैकि अच्छादक)      (b) many one into (बहुएक अनाच्छादक)

(4)

- (c) Many-one onto (बहुएक अच्छादक) (d) one-one into (एकैकि अनाच्छादक)
34. How many different matrices of unequal elements can be made by having the first 6 positive integer as element.  
 (प्रथम 6 धनात्मक पूर्णांक को लेते हुए असमान अवयव रखनेवाले कितने भिन्न-भिन्न आव्युह बनाये जा सकते हैं ?)  
 (a) 1880      (b) 1440      (c) 720      (d) 360
35. If  $A$  be a non-singular matrix of order  $3 \times 3$  then  $|\text{Adj } A| =$   
 (यदि  $A$  एक  $3 \times 3$  का व्युत्क्रमणीय आव्युह है तो  $|\text{Adj } A| =$ )  
 (a)  $3|A|$       (b)  $|A|$       (c)  $|A|^2$       (d)  $|A|^3$
36. If  $y = \log \sin x^2$ , then  $\frac{dy}{dx}$  at  $x = \sqrt{\frac{\pi}{2}}$  equal to  
 (यदि  $y = \log \sin x^2$ , तो  $x = \sqrt{\frac{\pi}{2}}$  पर  $\frac{dy}{dx}$  बराबर है ?)  
 (a) 0      (b) 1      (c)  $\frac{\pi}{4}$       (d)  $\sqrt{\pi}$
37. If  $f(x) = \sqrt{3} \sin x + \cos x$  then maximum value of  $f(x)$  is at what value of  $x$ .  
 (यदि  $f(x) = \sqrt{3} \sin x + \cos x$  है, तो  $f(x)$  का अधिकतम मान  $x$  के किस मान के लिए है ?)  
 (a)  $\frac{\pi}{6}$       (b)  $\frac{\pi}{2}$       (c)  $\frac{\pi}{3}$       (d)  $\frac{\pi}{4}$
38. A solution of the differential equation  $\left(\frac{dy}{dx}\right)^2 - x \frac{dy}{dx} + y = 0$  is  
 (अवकल समीकरण  $\left(\frac{dy}{dx}\right)^2 - x \frac{dy}{dx} + y = 0$  का एक हल है ?)  
 (a)  $y = 2$       (b)  $y = 2x$       (c)  $y = 2x - 4$       (d)  $y = 2x^2 - 4$
39. If (यदि)  $\vec{a} = 2\hat{i} - 5\hat{j} + \hat{k}$  and (और)  $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$  then (तो) –  
 (a)  $\vec{a} \cdot \vec{b} = 0$       (b)  $\vec{a} \cdot \vec{b} \neq 0$       (c)  $\vec{a} \cdot \vec{b} = -9$       (d)  $\vec{a} \perp \vec{b}$
40. If  $2x + 5y - 6z + 3 = 0$  be the equation of any plane then the equation of any plane parallel to the given plane is  
 (यदि किसी समतल का समीकरण  $2x + 5y - 6z + 3 = 0$  है तो इस समतल के समांतर किसी समतल का समीकरण है ?)  
 (a)  $3x + 5y - 6z + 3 = 0$       (b)  $2x - 5y - 6z + 3 = 0$   
 (c)  $2x + 5y - 6z + k = 0$       (d) None of these (इनमें से कोई नहीं)
41.  $\int_{-\pi/2}^{\pi/2} \sin^9 x dx =$   
 (a) -1      (b) 0      (c) 1      (d) None of these (इनमें से कोई नहीं)

(5)

*flame*

42. The chance of getting a doublet when two dice are rolled.

(दो पासों को फेंका जाता है तो एक द्विक प्राप्त करने की प्रायिकता है ?

- (a)  $\frac{2}{3}$       (b)  $\frac{1}{6}$       (c)  $\frac{5}{6}$       (d)  $\frac{5}{36}$

43. The slope of the normal to the curve  $y = x^2 + 3x + 4$  at (1, 1) is

(वक्र  $y = x^2 + 3x + 4$  के बिन्दु (1, 1) पर अभिलंब की प्रवणता है ?

- (a) 5      (b)  $-\frac{1}{5}$       (c) 8      (d)  $-\frac{1}{8}$

44. If (यदि)  $P(A) = \frac{1}{3}$ ,  $P(B) = \frac{1}{4}$ , and (और)  $P(A \cap B) = \frac{1}{5}$  then (तो)  $P\left(\frac{A}{B}\right) =$

- (a)  $\frac{1}{5}$       (b)  $\frac{2}{5}$       (c)  $\frac{3}{5}$       (d)  $\frac{4}{5}$

45. If (यदि)  $P(A) = 0.2$ ,  $P(B/A) = 0.3$  then (तो)  $P(A \cap B) =$

- (a) 0.9      (b) 0.06      (c) 0.8      (d) None of these (इनमें से कोई नहीं)

$$46. \int_{-1}^1 |x| dx =$$

- (a)  $\frac{1}{2}$       (b) 1      (c) 1      (d) None of these (इनमें से कोई नहीं)

$$47. \int \frac{1}{e^x + e^{-x}} dx =$$

- (a)  $\cot^{-1}(e^x) + k$       (b)  $\cot^{-1}(e^{-x}) + k$       (c)  $\tan^{-1}(e^x) + k$       (d)  $\tan^{-1}(e^{-x}) + k$

$$48. \int_0^3 x(3-x)^{3/2} dx =$$

- (a)  $\frac{108\sqrt{3}}{35}$       (b)  $-\frac{108\sqrt{3}}{35}$       (c)  $-\frac{54\sqrt{3}}{35}$       (d) None of these (इनमें से कोई नहीं)

49. If (यदि)  $A = \begin{vmatrix} i & 0 \\ 0 & i \end{vmatrix}$  then (तो)  $A^2 =$

- (a)  $\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$       (b)  $\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$       (c)  $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$       (d)  $\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$

50. If  $2 \begin{vmatrix} x & 5 \\ 3 & y \end{vmatrix} = \begin{vmatrix} 4 & 10 \\ 6 & 6 \end{vmatrix}$  then the value of  $x$  &  $y$  are

(यदि  $2 \begin{vmatrix} x & 5 \\ 3 & y \end{vmatrix} = \begin{vmatrix} 4 & 10 \\ 6 & 6 \end{vmatrix}$  तो  $x$  और  $y$  का मान है ?)

- (a)  $x=2, y=3$       (b)  $x=3, y=2$       (c)  $x=2, y=2$       (d)  $x=3, y=3$

(6)

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## SECTION ( खण्ड )-II

### Non-Objective Questions ( गैर वस्तुनिष्ट प्रश्न )

**Time : [2 Hrs + 5 Min. (Extra)]**

**Full Marks : 50**

**समय : 2 घंटा + 5 मिनट ( अतिरिक्त )**

**पूर्णांक : 50**

Question number 1 to 22 are of short Answer type. Each question carry 2 marks. Answer any 15 Question.

[ प्रश्न संख्या 1 से 22 तक लघुउत्तरीय प्रकार के हैं। प्रत्येक के लिए 2 अंक निर्धारित हैं। किन्हीं 15 प्रश्नों के उत्तर दें। ]

$$15 \times 2 = 30$$

1. If  $\vec{a} = 4\hat{i} + 3\hat{j} + 2\hat{k}$  and  $\vec{b} = 3\hat{i} + 2\hat{k}$  then find  $|\vec{b} \times 2\vec{a}|$

( यदि  $\vec{a} = 4\hat{i} + 3\hat{j} + 2\hat{k}$  और  $\vec{b} = 3\hat{i} + 2\hat{k}$  तो  $|\vec{b} \times 2\vec{a}|$  ज्ञात करें ? )

2. If (यदि)  $y = \cos^{-1} \frac{1-x^2}{1+x^2}$  than find (तो ज्ञात करें)  $\frac{dy}{dx}$ .

3. Solve (हल करें) :  $\sin^{-1} \frac{2a}{1+a^2} + \cos^{-1} \frac{1-b^2}{1+b^2} = 2\tan^{-1} x$

4. Prove that the determinant  $\begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & 2 \end{vmatrix}$  is independent of  $\theta$ .

( सिद्ध करें कि सारणिक  $\begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & 2 \end{vmatrix}$ ,  $\theta$  से स्वतंत्र है? )

5. Integrate : (समाकलन करें) :  $\int e^x \cdot \cos(e^x) dx$

6. If (यदि)  $x \cos y = \sin(x+y)$  than find (तो ज्ञात करें)  $\frac{dy}{dx}$

7. If  $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$  and  $B = [1, 0, 4]$  than verify  $(A \cdot B)' = B'A'$ .

( यदि  $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$  और  $B = [1, 0, 4]$  तो  $(A \cdot B)' = B'A'$  की जाँच करें। )

8. Solve differential equation ( अवकल समीकरण को हल करें। )  $\frac{dy}{dx} = 1 + x + y + xy$

9. If  $P(A) = 0.4$ ,  $P(B) = 0.8$ ,  $P(B/A) = 0.6$  find  $P(A/B)$  and  $P(A \cup B)$

( यदि  $P(A) = 0.4$ ,  $P(B) = 0.8$ ,  $P(B/A) = 0.6$  है, तो  $P(A/B)$  और  $P(A \cup B)$  ज्ञात करें। )

10. If  $f(x) = \frac{x^2}{|x|}, x \neq 0$

$$= 0, x = 0$$

then test the continuity of  $f(x)$  at  $x = 0$

(यदि  $f(x) = \frac{x^2}{|x|}, x \neq 0$ )

$= 0, x = 0$  तो  $x = 0$  पर  $f(x)$  की सतता की जाँच करें।

11. Integrate (समाकलन करें) :  $\int_1^e \frac{e^x}{x} (1 + x \log x) dx$

12. Verify Rolle's theorem for  $f(x) = x^2 - 5x + 6$  in  $[2, 3]$ .

(अन्तराल  $[2, 3]$  में  $f(x) = x^2 - 5x + 6$  के लिए राँले प्रमेय की जाँच करें।)

13. Prive that  $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$  is always an increasing function in  $\left(0, \frac{\pi}{4}\right)$ .

(सिद्ध करें कि  $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$  अन्तराल  $\left(0, \frac{\pi}{4}\right)$  में सदा एक वर्धमान फलन है।

14. Two dice are thrown. Find the probability that the number appeared has a sum 8 if it is known that the second dice always exhibits 4?

(दो पासे फेंकने के क्रम में उस पर आए अंकों का योग 8 होने की प्रायिकता ज्ञात कीजिए जबकि मालूम हो कि दूसरे पासे पर हमेशा 4 आता है।)

15. Let A & B are independent event with  $P(A) = 0.3$  and  $P(B) = 0.4$ . Find (i)  $P(A \cap B)$  (ii)  $P(A / B)$   
(माना कि A और B स्वतंत्र घटना है तथा  $P(A) = 0.3, P(B) = 0.4$ . तो (i)  $P(A \cap B)$  (ii)  $P(A / B)$  ज्ञात करें।)

16. Let \* be a binary operation on N, N being set of natural number defined by  $a * b = a^b$  for all  $a, b \in N$ .

Verfy \* is associative and commutative on N.

(माना कि '\*' N पर एक द्विआधारी संक्रिया है, जहाँ N प्राकृतिक संख्याओं का समुच्चय है। \* को  $a * b = a^b \quad \forall a, b \in N$  द्वारा परिभाषित किया गया है।)

\* के सहचर्य और क्रमविनिमय होने की जाँच करें।

17. Prove that (सिद्ध करें)

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$$

18. Find equation of plane through the point  $(3, 4, -1)$  which is parallel to plane.

$$\vec{r} \cdot (2\hat{i} - 3\hat{j} + 5\hat{k}) + 7 = 0$$

(बिन्दु  $(3, 4, -1)$  से गुजरने वाली और समतल  $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 5\hat{k}) + 7 = 0$  के समानान्तर समतल का समीकरण ज्ञात करें।)

(8)

*Syamal*

19. Find the angle between the lines (रेखाओं के बीच का कोण ज्ञात करें)

$$\vec{r} = 4\hat{i} - \hat{j} + \lambda(\hat{i} + 2\hat{j} - 2\hat{k}) \text{ and}$$

$$\vec{r} = \hat{i} - \hat{j} + 2\hat{k} - \mu(2\hat{i} + 4\hat{j} - 4\hat{k})$$

20. If  $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$  then find  $A^3$

(यदि  $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$  तो  $A^3$  का मान ज्ञात करें।)

21. If  $R = \{(x, y) : x, y \text{ are integers such that } x - y \text{ is divisible by 5}\}$  Show that  $R$  is Reflective and symmetric.

(यदि  $R = \{(x, y) : x, y \text{ पूर्णांक हैं और } x - y, 5 \text{ से विभाजित है}\}$  दिखाइए कि  $R$  स्वतुल्य और सममित है।

22. Evaluate (ज्ञात करें) :  $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

प्रश्न संख्या 23 से 26 दीर्घउत्तरीय कोटि के हैं। प्रत्येक के लिए 5 अंक निर्धारित है। प्रत्येक प्रश्न या उसके विकल्प में से किसी एक का उत्तर दें।

Question number 23 to 26 are of long answer type. Each question carry 5 marks. Answer every question or alternative of it. [4 × 5]

23. Integrate (समाकलन करें) :  $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x}) dx$

Or, (या)

Find the area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(दीर्घ वृत्त  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  का क्षेत्रफल ज्ञात करें।)

24. If the straight line  $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$  and  $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$  intersects at a point, then find the value of Integer  $k$ .

(यदि सरल रेखा  $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$  तथा  $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$  एक दूसरे को प्रतिच्छेद करती है तो  $k$  का मान ज्ञात करें।)

Or, (या)

Find  $\lambda$  when the projection of  $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$  on  $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$  is 4 unit.

(9)

(यदि  $\vec{a} = \lambda \hat{i} + \hat{j} + 4\hat{k}$  का प्रक्षेप  $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$  पर 4 ईकाई है तो  $\lambda$  का मान ज्ञात करें।

25. Solve (हल करें) :  $x \log x \frac{dy}{dx} + y = \frac{2}{y} \log x$

Or, (या)

If (यदि)  $x = a \left( \frac{1+t^2}{1-t^2} \right)$  and (और)  $y = \frac{2t}{1-t^2}$  find (ज्ञात करें)  $\frac{dy}{dx}$ .

26. Maximum (अधिकतमीकरण)  $z = 5x + 3y$

Subjected to  $3x + 5y \leq 15$

(जबकि)  $5x + 2y \leq 10$

$x, y \geq 0$

Or, (या)

Minimize (न्यूनतमीकरण)  $z = -3x + 4y$

subjected to जबकि  $x + 2y \leq 8$

$3x + 2y \leq 12$

$x \geq 0, y \geq 0$

(10)

# SOLUTION Answer Key

## Objective Question :

1. (b)	2. (a)	3. (b)	4. (b)	5. (c)	6. (a)	7. (a)	8. (b)	9. (c)	10. (b)
11. (b)	12. (c)	13. (b)	14. (b)	15. (b)	16. (d)	17. (d)	18. (b)	19. (b)	20. (b)
21. (a)	22. (b)	23. (c)	24. (b)	25. (d)	26. (b)	27. (a)	28. (a)	29. (a)	30. (b)
31. (a)	32. (c)	33. (b)	34. (a)	35. (c)	36. (d)	37. (c)	38. (c)	39. (c)	40. (c)
41. (b)	42. (b)	43. (b)	44. (d)	45. (b)	46. (b)	47. (c)	48. (a)	49. (b)	50. (b)

## Solution to Short Answer type question :

1. We have  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$\therefore \vec{2a} = \hat{8i} + \hat{6j} + \hat{4k}$$

and  $\vec{b} = \hat{3i} + \hat{2k} = \hat{3i} + \hat{0j} + \hat{2k}$

$$\begin{aligned} \therefore \vec{b} \times \vec{2a} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 2 \\ 8 & 6 & 4 \end{vmatrix} = \hat{i}(0-12) - \hat{j}(12-16) + \hat{k}(18-0) \\ &= -12\hat{i} + 4\hat{j} + 18\hat{k} \end{aligned}$$

$$\therefore \left| \vec{b} \times \vec{2a} \right| = \sqrt{12^2 + 4^2 + 18^2} = \sqrt{484} = 2\sqrt{121} = 22$$

2. Putting  $x = \tan \theta$ , we get

$$y = \cos^{-1} \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos^{-1} \cos 2\theta = 2\theta = 2 \tan^{-1} x$$

$$\therefore \frac{dy}{dx} = 2 \times \frac{1}{1+x^2} = \frac{2}{1+x^2}$$

3. Since,  $\sin^{-1} \frac{2a}{1+a^2} = 2\tan^{-1} a$

and  $\cos^{-1} \frac{1-b^2}{1+b^2} = 2\tan^{-1} b$

$\therefore$  given problem can be written as  
 $\tan^{-1} a + \tan^{-1} b = \tan^{-1} x$

$$\Rightarrow \tan^{-1} \frac{a+b}{1-ab} = \tan^{-1} x$$

(11)



$$\therefore x = \frac{a-b}{1-ab}$$

4. Expanding along the first row, we get

$$\begin{aligned}\Delta &= x(-x^2 - 1) - \sin\theta (-x \sin\theta - \cos\theta) + \cos\theta (-\sin\theta + x \cos\theta) \\ &= -x^3 + x + x \sin^2\theta + \sin\theta \cdot \cos\theta - \sin\theta \cdot \cos\theta + x \cos^2\theta \\ &= -x^3 + x + x(\sin^2\theta + \cos^2\theta) \\ &= -x^3 + 2x\end{aligned}$$

Hence, given determinant is independent of  $\theta$ .

5. Let  $e^x = z$

$$\therefore e^x \cdot dx = dz$$

$$\therefore I = \int e^x \cdot \cos(e^x) dx = \int \cos z \cdot dz = \sin z + k = \sin e^x + k$$

6. Given,  $x \cos y = \sin(x+y)$

Differentiating both sides w.r.t. 'x', we get

$$\begin{aligned}x(-\sin y) \cdot \frac{dy}{dx} + \cos y &= \cos(x+y) \left[ 1 + \frac{dy}{dx} \right] \\ \Rightarrow -x \sin y \cdot \frac{dy}{dx} + \cos y &= \cos(x+y) + \cos(x+y) \cdot \frac{dy}{dx} \\ \Rightarrow \frac{dy}{dx} [\cos(x+y) + x \sin y] &= \cos y - \cos(x+y) \\ \therefore \frac{dy}{dx} &= \frac{\cos y - \cos(x+y)}{\cos(x+y) + x \sin y}\end{aligned}$$

7. Given,  $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}_{3 \times 1}$  and  $B = [1, 0, 4]_{1 \times 3}$

$$\therefore AB = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \cdot [1, 0, 4] = \begin{bmatrix} 3 & 0 & 12 \\ 5 & 0 & 20 \\ 2 & 0 & 8 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

$$\text{Now, } A' = [3, 5, 2] \text{ and } B' = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} \cdot [3, 5, 2] = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

$$\therefore (AB)' = B'A'$$

(12)

8. Given differential equation is

$$\begin{aligned}\frac{dy}{dx} &= 1 + x + y + xy = (1+x) + y(1+x) \\ &= (1+x)(1+y)\end{aligned}$$

$$\text{or, } \frac{dy}{1+y} = (1+x)dx$$

Integrating, we get

$$\int \frac{dy}{1+y} = \int (1+x)dx$$

$$\Rightarrow \log(1+y) = x + \frac{x^2}{2} + k$$

which is required solution.

$$9. \because P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right) = 0.4 \times 0.6 = 0.24$$

$$\text{Now, } P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.8} = 0.3$$

$$\text{and } P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.8 - 0.2 = 0.96$$

10. By definition,

$$|x| = x, x > 0$$

$$= 0, x = 0$$

$$= -x, x < 0$$

$$\therefore f(x) = \frac{x^2}{|x|} = x, x > 0$$

$$= \frac{0}{0} \text{ is undefined for } x = 0$$

$$= -x, x < 0$$

Since,  $f(x)$  is not defined at  $x = 0$ , therefore the given function is not continuous at  $x = 0$

$$\begin{aligned}11. \int \frac{e^x}{x} (1 + x \log x) dx &= \int e^x \left( \frac{1}{x} + \log x \right) dx \\ &= \int e^x (f'(x) + f(x)) dx \text{ where } f(x) = \log x \\ &= e^x \cdot f(x) = e^x \cdot \log x\end{aligned}$$

$$\therefore \int_1^e \frac{e^x}{x} (1 + x \log x) dx = \left[ e^x \cdot \log x \right]_1^e = e^e \cdot \log e^e - e \log 1 = e^e$$

12. Given,  $f(x) = x^2 - 5x + 6$  in  $[2, 3]$

since,  $f(x)$  being a polynomial function in  $x$ .

So,  $f(x)$  is differentiable and continuous everywhere..

(13)



So,  $f(x)$  is continuous in  $[2, 3]$  and derivable in  $(2, 3)$

Also,  $f(2) = 0$  and  $f(3) = 0$

$$\therefore f(2) = f(3)$$

Hence, all the conditions of Rolle's theorem are satisfied for  $f(x)$  in  $[2, 3]$

So, there exist  $C \in (2, 3)$  such that  $f'(C) = 0$

Here,  $f(x) = 2x - 5$

$$\text{So, } f(C) = 0 \Rightarrow 2C - 5 = 0 \Rightarrow C = \frac{5}{2}$$

$$\therefore C = \frac{5}{2} \in (2, 3) \text{ such that } f(C) = 0$$

Hence, Rolle's theorem verified.

13. Given that,  $f(x) = \tan^{-1}(\sin x + \cos x)$ ,  $x > 0$

$$\begin{aligned}\therefore f'(x) &= \frac{1}{1+(\sin x + \cos x)^2} \cdot (\cos x - \sin x) \\ &= \frac{\cos x}{1+(\sin x + \cos x)^2} \cdot (1 - \tan x) > 0, \quad \forall x \in \left(0, \frac{\pi}{4}\right)\end{aligned}$$

$$\text{Hence, } f'(x) > 0 \text{ in } \left(0, \frac{\pi}{4}\right)$$

Therefore  $f(x)$  is an increasing function in  $\left(0, \frac{\pi}{4}\right)$

14. Let A be an event of always 4 exhibits on second dice.

$$\text{i.e. } A = \{(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)\}$$

$$n(A) = 6$$

and B be an event of the numbers appeared has a sum of 8.

$$\text{i.e. } B = \{(4, 4)\}$$

$$\therefore A \cap B = \{(4, 4)\}$$

$$\therefore n(A \cap B) = 1$$

$$\text{Hence, } P\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)} = \frac{1}{6}$$

15. Since, A and B are independent events

$$\text{So, (i) } P(A \cap B) = P(A) \cdot P(B)$$

$$= 0.3 \times 0.4 = 0.12$$

$$\text{(ii) } P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.4} = 0.3 = \frac{3}{10}$$

16. Given, binary operation is  $a * b = a^b$

$$\text{Let, } 2, 3 \in \mathbb{N}$$

$$\text{So, } 2 * 3 = 2^3 = 8 \text{ and } 3 * 2 = 3^2 = 9$$

$$\therefore 2 * 3 \neq 3 * 2$$

So, '\*' is not commutative on  $\mathbb{N}$ .

(14)

*Harper*

Also,  $2 * (2 * 3) = 2 * 2^3 = 2 * 8 = 2^8 = 256$

and  $(2 * 2) * 3 = 2^2 * 3 = 4 * 3 = 4^3 = 64$

So,  $2 * (2 * 3) \neq (2 * 2) * 3$

So, \* is not associative on 'N'

17. LHS =  $\vec{a} - \vec{b}$   $\times$   $(\vec{a} + \vec{b})$

$$= \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b} \quad (\text{Distributive law})$$

$$= \vec{a} \times \vec{b} - \vec{b} \times \vec{a} \quad \left[ \because \vec{a} \times \vec{a} = 0 \right]$$

$$= (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{b})$$

$$= 2(\vec{a} \times \vec{b})$$

18. Let equation of any plane parallel to the given plane is given by

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = d \quad \dots\dots\dots (i)$$

If this plane passes through the point  $(3, 4, -1)$  whose position vector is  $\hat{3i} + \hat{4j} - \hat{k}$

Then,  $(\hat{3i} + \hat{4j} - \hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = d$

$\Rightarrow 3.1 + 4.(-3) + (-1).5 = d$

$\Rightarrow d = -11$

By (i) we get

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) + 11 = 0 \text{ which is the required equation of the plane.}$$

19. Given line be:-

$$\vec{r} = \hat{4i} - \hat{j} + \lambda(\hat{i} + 2\hat{j} - 2\hat{k}) \quad \dots\dots\dots (i)$$

$$\text{and } \vec{r} = \hat{i} + \hat{j} + 2\hat{k} - \mu(\hat{2i} + \hat{4j} - \hat{4k}) \quad \dots\dots\dots (ii)$$

The first line is parallel to the vector  $\vec{b}_1 = \hat{i} + 2\hat{j} - 2\hat{k}$  and second line is parallel to the vector  $\vec{b}_2 =$

$\hat{2i} + \hat{4j} - \hat{4k}$ . If ' $\theta$ ' be the angle between the lines, then  $\theta$  is also the angle between the vector  $\vec{b}_1$  &  $\vec{b}_2$ .

$$\text{So, } \cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} = \frac{1.2 + 2.4 + (-2)(-4)}{\sqrt{1^2 + 2^2 + (-2)^2} \sqrt{2^2 + 4^2 + (-4)^2}} = \frac{18}{\sqrt{9} \sqrt{36}}$$

$$= \frac{18}{3 \times 6} = 1 \quad \therefore \theta = 0$$

(15)

*[Signature]*

Hence, angle between the line is 0

20. Given  $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$$\text{Then, } A^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1+1 & -1+(-1) \\ -1+(-1) & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\text{So, } A^3 = A^2 \cdot A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2+2 & -2-2 \\ -2-2 & 2+2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

21. Let  $x, y, z \in \mathbb{Z}$ , then

(i)  $x - x$  is divisible by 5  $\forall x \in \mathbb{Z}$

$\therefore R$  is reflexive

(ii) If  $x - y$  is divisible by 5, then  $y - x$  is also divisible by 5  $\forall x, y \in \mathbb{Z}$

$\therefore R$  is symmetric

22.  $\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 0 \cdot \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} + 0 \cdot \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$   
 $= 1(1 - 0) + 0 + 0 = 1$

### Long Answer Type Question (दीर्घ उत्तरीय प्रश्न)

$$23. \int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) dx = \int_0^{\frac{\pi}{2}} \left( \frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} \right) dx = \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sqrt{\sin x \cdot \cos x}} dx$$

$$= \sqrt{2} \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sqrt{2 \sin x \cdot \cos x}} dx = \sqrt{2} \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sqrt{1 - (1 - 2 \sin x \cdot \cos x)}} dx$$

$$= \sqrt{2} \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}} dx$$

Put  $\sin x - \cos x = t \Rightarrow (\cos x + \sin x) dx = dt$

when  $x = 0, t = -1$  and when  $x = \frac{\pi}{2}, t = 1$

$$\text{So, } \int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) dx = \sqrt{2} \int_{-1}^1 \frac{dt}{\sqrt{1-t^2}} = \sqrt{2} [\sin^{-1} t]_{-1}^1$$

$$= \sqrt{2} [\sin^{-1} 1 - \sin^{-1} (-1)] = \sqrt{2} \left( \frac{\pi}{2} - \left( -\frac{\pi}{2} \right) \right) = \sqrt{2} \cdot \pi$$

OR,

(16)

Given equation of the ellipse is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\text{So, } y = \sqrt{b^2 \left(1 - \frac{x^2}{a^2}\right)} = \frac{b}{a} \sqrt{a^2 - x^2} \quad (\because y \geq 0 \text{ for part AB})$$

$$\text{Now area AOBA} = \int_0^a y dx = \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} dx$$

$$\begin{aligned} &= \frac{b}{a} \left[ \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a \\ &= \frac{b}{a} \left[ 0 + \frac{a^2}{2} \sin^{-1} 1 - 0 - 0 \right] \\ &= \frac{b}{a} \times \frac{a^2}{2} \cdot \frac{\pi}{2} = \frac{\pi ab}{4} \end{aligned}$$

$$\therefore \text{Area of ellipse} = 4 \times \frac{\pi ab}{4} = \pi ab \text{ sq. unit}$$

$$24. \text{ Given lines are } \frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3} = \lambda \quad \dots\dots(i)$$

$$\text{and } \frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2} = \mu \quad \dots\dots(ii)$$

Any point on line (i) is  $(k\lambda + 1, 2\lambda + 2, 3\lambda + 3)$  and any point on line (ii)  $(3\mu + 2, \mu k + 3, 2\mu + 1)$   
If these line intersect, these points must coincide for some value of  $\lambda$  and  $\mu$ .

$$\text{So, } \lambda k + 1 = 3\mu + 2, 2\lambda + 2 = \mu k + 3, 3\lambda + 3 = 2\mu + 1$$

$$\Rightarrow \lambda k - 3\lambda - 1 = 0, 2\lambda - \mu k - 1 = 0, 3\lambda - 2\mu + 2 = 0$$

taking first two, we get

$$\frac{\lambda}{3-k} = \frac{\mu}{-2+k} = \frac{1}{-k^2+6} \Rightarrow \lambda = \frac{3-k}{6-k^2}, \mu = \frac{k-2}{6-k^2}$$

Putting these values of  $\lambda$  &  $\mu$  in  $3\lambda - 2\mu + 2 = 0$

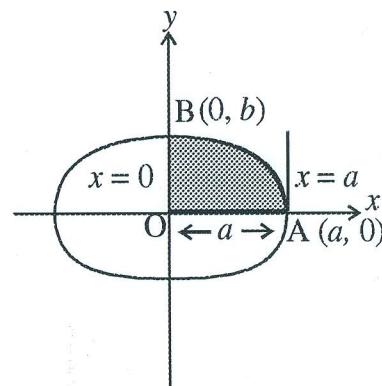
$$3\left(\frac{3-k}{6-k^2}\right) - 2\left(\frac{k-2}{6-k^2}\right) + 2 = 0 \Rightarrow 9 - 3k - 2k + 4 + 12 - 2k^2 = 0$$

$$\Rightarrow 2k^2 + 5k - 25 = 0 \Rightarrow 2k(k+5) - 5(k+5) = 0$$

$$\Rightarrow (2k-5)(k+5) = 0 \Rightarrow k = \frac{5}{2}, -5$$

$k = -5$  satisfy both the equations. Hence  $k = -5$  is required value.

OR,



(17)

*[Handwritten signature]*

The projection of  $\vec{a}$  on  $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$$\text{Now, } \vec{a} \cdot \vec{b} = (\hat{i} + \hat{j} + \hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k}) = 2\lambda + 18$$

$$\text{and } |\vec{b}| = \sqrt{2^2 + 6^2 + 3^2} = \sqrt{49} = 7$$

But it is given that  $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 4$

$$\therefore \frac{2\lambda + 18}{7} = 4$$

$$\therefore 2\lambda = 28 - 18 = 10 \therefore \lambda = 5$$

25. Given differential equation may be written as

$$\frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x^2} \quad (\text{which is linear diff. equation})$$

$$\text{Here, } P = \frac{1}{x \log x} \text{ and } Q = \frac{2}{x^2}$$

$$\text{Now, } \int P dx = \int \frac{1}{x \log x} dx \quad [\text{Put } \log x = z \Rightarrow \frac{1}{x} dx = dz]$$

$$= \int \frac{1}{z} dz = \log z = \log(\log x)$$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{\log(\log x)} = \log x$$

Hence, solution is given by

$$\begin{aligned} y \times \log x &= \int \frac{2}{x^2} \cdot \log x dx = 2 \int x^{-2} \cdot \log x dx \\ &= 2 \left[ (\log x) \cdot \frac{x^{-2+1}}{-2+1} - \int \frac{1}{x} \left( -\frac{1}{x} \right) dx \right] \\ &= 2 \left[ -\frac{1}{x} \log x + \int \frac{1}{x^2} dx \right] \\ &= 2 \left[ -\frac{1}{x} \log x - \frac{1}{x} \right] + k = -\frac{2}{x} (\log x + 1) + k \end{aligned}$$

(18)

*[Signature]*

**OR,**

$$\text{Here, } x = a \left( \frac{1+t^2}{1-t^2} \right) = a \left( -1 + \frac{2}{1-t^2} \right)$$

$$\therefore \frac{dx}{dt} = a \left( 0 + 2 \times \frac{-1}{(1-t^2)^2} \cdot (-2t) \right) = \frac{4at}{(1-t^2)^2}$$

$$\text{and } y = \frac{2t}{1-t^2}$$

$$\therefore \frac{dy}{dt} = \frac{(1-t^2).2 - 2t.(-2t)}{(1-t^2)^2} = \frac{2(1+t^2)}{(1-t^2)^2}$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2(1+t^2)}{(1-t^2)^2} \times \frac{(1-t^2)^2}{4at} = \frac{(1+t^2)}{2at}$$

26. First we draw the lines  $3x + 5y = 15$  and  $5x + 2y = 10$  to determine the feasible region.

The shaded region OCPBO is the feasible region. vertices of the feasible region are O(0, 0), C(2, 0),

$$P\left(\frac{20}{19}, \frac{45}{19}\right) \text{ and } B(0, 3)$$

Now, we calculate  $z = 5x + 3y$  at each vertex

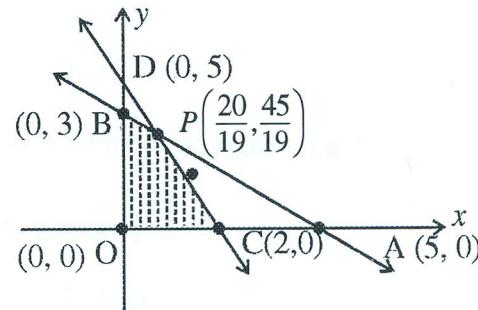
$$\text{At } (0, 0) z = 0$$

$$\text{At } (2, 0) z = 10$$

$$\text{At } \left(\frac{20}{19}, \frac{45}{19}\right), z = \frac{235}{19} = 12.36$$

$$\text{At } (0, 3), z = 9$$

Hence, maximum value of  $z$  is 12.36 at  $\left(\frac{20}{19}, \frac{45}{19}\right)$ .



**OR,**

We draw the lines  $x + 2y = 8$  and  $3x + 2y = 12$  to determine the feasible region. The shaded region OCEBO is the feasible region. vertices of the feasible region are O(0, 0), C(4, 0), E(2, 3) and B(0, 4).

Now we calculate  $z = -3x + 4y$  at each vertex.

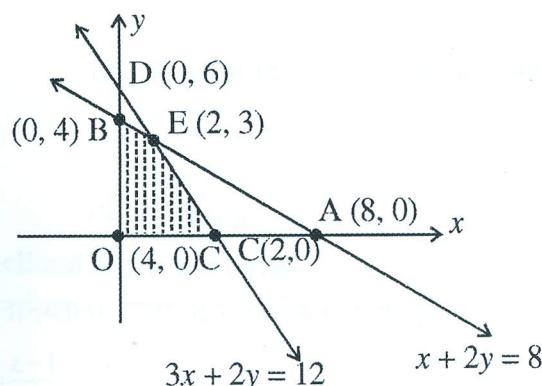
$$\text{at } (0, 0), z = 0$$

$$\text{at } (4, 0), z = -12$$

$$\text{at } (2, 3), z = 6$$

$$\text{at } (0, 4), z = 16$$

Hence minimum value of  $z$  is -12 at (4, 0)



(19)

*Singh*