SET (प्रारूप)–5

SECTION (खण्ड)–I

Objective Questions (वस्तुनिष्ट प्रश्न)

Time : [1 Hrs **Full Marks : 50**

समय : 1 घंटा **पूर्णांक : 50**

From Question no. 1 to 50 these is only one correct answer for each question. You have to mark the correct option from the given options.

[प्रश्न संख्या 1 से 50 तक प्रत्येक प्रश्न के केवल एक उत्तर सही है। प्रत्येक प्रश्न के सही उत्तर को चुनकर उत्तर तालिका में चिह्नित करें] **[50 × 1]**

1. The solution of LPP. maximize $z = 3x + 2y$

Subjected to, $x + 2y \leq 10$

$3x + y \leq 15$

$x, y, \geq 0$

lies in which quadrant.

रैखिक प्रोग्रामिंग समस्या अधिकतम $z = 3x + 2y$

$x + 2y \leq 0$

जबकि $3x + y \leq 15$

$x, y, \geq 0$

का हल किस पाद में होगा ?

(a) first (प्रथम) (b) second (द्वितीय) (c) Third (तृतीय) (d) Fourth (चतुर्थ)

2. The mean and varience of a bionomial distribution X are 2 and 1 respectively then what is the value of P $(X \geq 1)$

(एक द्विपद वितरण X का माध्य और प्रसरण क्रमशः 2 और 1 है । तो P $(X \geq 1)$ का मान क्या है?

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{16}$ (d) None of these (इनमें से कोई नहीं)

3. $\frac{d(a^x)}{dx} =$

(a) $a^x \log a$ (b) $\frac{a^x}{\log a}$ (c) $a^{x\log a}$ (d) None of these (इनमें से कोई नहीं)

4. If binary operation '*' is defined as $a * b = a^2 + b^2$ then $(1 * 2) * 6$ equal to

(यदि द्विपद संक्रिया '*', $a * b = a^2 + b^2$ के द्वारा परिभाषित है, तो $(1 * 2) * 6$ का मान बराबर है ।

(a) 12 (b) 28 (c) 61 (d) 9

5. $\cos(\sin^{-1}x + \cos^{-1}x)$

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

6. If $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then value of $\frac{dy}{dx}$ is

(यदि $\sqrt{x}+\sqrt{y}=\sqrt{a}$ तो $\frac{dy}{dx}$ का मान है ?)

(a) $-\frac{\sqrt{x}}{\sqrt{y}}$ (b) $-\frac{1}{2}\sqrt{\frac{y}{x}}$ (c) $-\sqrt{\frac{y}{x}}$ (d) None of these (इनमें से कोई नहीं)

7. **The principal value of** $\tan^{-1}(-\sqrt{3})$

$\tan^{-1}(-\sqrt{3})$ **का मुख्य मान है ?**

(a) $\frac{2\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $-\frac{\pi}{3}$ (d) None of these (इनमें से कोई नहीं)

8. $\frac{d(e^{\sin\sqrt{x}})}{dx} =$

(a) $e^{\sin\sqrt{x}}.\cos\sqrt{x}$ (b) $\frac{e^{\sin\sqrt{x}}.\cos\sqrt{x}}{2\sqrt{x}}$ (c) $\frac{e^{\sin\sqrt{x}}}{2\sqrt{x}}$ (d) $e^{\sin\sqrt{x}}$

9. **Magnitude of the vector** $\hat{i}\times(\hat{j}+\hat{k})+\hat{j}\times(\hat{k}+\hat{i})+\hat{k}\times(\hat{k}+\hat{j})$ **is**

(सदिश $\hat{i}\times(\hat{j}+\hat{k})+\hat{j}\times(\hat{k}+\hat{i})+\hat{k}\times(\hat{k}+\hat{j})$ **का परिमाप है ?)**

(a) 0 (b) 1 (c) –1 (d) $|\hat{i}\times(\hat{j}\times\hat{k}|$

10. $\int_0^1 x\sqrt{x}\,dx =$

(a) 12. 8 (b) 12.4 (c) 7 (d) 9

11. **Which of the following is general form of LPP.**

(निम्न में से LPP का सामान्य प्रतिरूप है ?)

(a) maximize $z = cx$ subjected to $AX \geq, =, \geq B, x \geq 0$

(b) minimize $z = cx$ subjected to $AX \leq, =, \geq B, x \leq, 0$

(c) Both (a) and (b) are true ((a) और (b) दोनों सत्य है

(d) None of these (इनमें से कोई नहीं)

12. **If (यदि)** $y = 9^x$ **then (तो)** $\frac{d^2y}{dx^2} =$

(a) $9^x.\log 9$ (b) $9^x.(\log 9)^2$ (c) $(9x)^2.\log 9$ (d) None of these (इनमें से कोई नहीं)

13. $\int a^{3x+3}\,dx (a>0) =$

(a) $a^3.\frac{a^{3x}}{3\log a}+k$ (b) $a^2.\frac{a^{3x}}{3\log a}+k$ (c) $\frac{a^{3x}}{\log a}+k$ (d) None of these (इनमें से कोई नहीं)

14. **Madulus of vector** $2\hat{i}-7\hat{j}-3\hat{k}$ **is**

(सदिश $2\hat{i}-7\hat{j}-3\hat{k}$ **का मापांक है ?)**

(a) $\sqrt{12}$ (b) $\sqrt{50}$ (c) $\sqrt{60}$ (d) $2\sqrt{5}$

15. **An objective function in general LPP is**
(सामान्य रैखिक प्रोग्रामिंग में उद्देश्य फलन होता है ?)
(a) Linear function (रैखिक फलन) (b) Non-Linear function (अरैखिक फलन)
(c) constant function (अचर फलन) (d) None of these (इनमें से कोई नहीं)

16. **The principal value of $\cos^{-1}\left(\frac{1}{-\sqrt{2}}\right)$ is**
($\cos^{-1}\left(\frac{1}{-\sqrt{2}}\right)$ का मुख्य मान है ?)
(a) $\frac{-2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{4}$ (d) $\frac{-\pi}{4}$

17. **Which of the following is true ?**
(निम्न में कौन-सा सत्य है ?)
(a) $(AB)^{-1} = A^{-1}B^{-1}$ (b) $(AB)^{-1} = B^{-1}A^{-1}$
(c) $(AB)^{-1} = (BA)^{-1}$ (d) None of these (इनमें से कोई नहीं)

18. **The order and degree of the differential equation $\frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}$ are respectively**
(अवकल समीकरण $\frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}$ के कोटि एवं घात क्रमशः है ?)
(a) $\frac{3}{2}$, 2 (b) 2, 2 (c) 2, $\frac{3}{2}$ (d) 3, 4

19. $\int \frac{dx}{a^2+x^2} =$
(a) $\log(x+\sqrt{x^2+a^2}$ (b) $\log(x-\sqrt{x^2+a^2}$
(c) $\log(x+\sqrt{x^2-a^2}$ (d)None of these (इनमें से कोई नहीं)

20. **If $f : R \to R$ be a relation given by $f = \{(a, a), (b, b), (c, c)\}$ then which is true for 'f'**
(यदि $f : R \to R$ एक सम्बन्ध है और $f = \{(a, a), (b, b), (c, c)\}$ तो 'f' के सम्बन्ध में कौन सत्य है ?)
(a) Reflexive (स्वतुल्य) (b) Symmetric (सममित)
(c) (a) and (b) both ((a) और (b) दोनों) (d) None of these (इनमें से कोई नहीं)

21. **The plane $x = 0$ and $y = 0$ is**
(तल $x = 0$ और $y = 0$ है ?)
(a) Parallel (समानान्तर) (b) perpendicular (लम्बवत)
(c) Intersect at x-axis (x-अक्ष पर कटती है)
(d) None of these (इनमें से कोई नहीं)

22. $\frac{d}{dx}(\sin^{-1}x + \cos^{-1}x) =$

(a) 1 (b) $\frac{\pi}{2}$ (c) 0 (d) None of these (इनमें से कोई नहीं)

23. **The direction ratio of the line joining the points A (2, – 4, 5) and B (1, – 1, 3) are**
(बिन्दु A (2, – 4, 5) और (1, – 1, 3) को मिलाने वाली रेखा का दिक् अनुपात है ?)
(a) (1, – 3, 2) (b) (–3, 1, 2) (c) (2, 1, –3) (d) None of these (इनमें से कोई नहीं)

24. **If (यदि) $y = \tan^2 x$, then (तो) $\frac{dy}{dx}$ =**
(a) $x^3 . \cos(x^3)$ (b) $\sec^2 x$ (c) $2\tan x . \sec^2 x$ (d) None of these (इनमें से कोई नहीं)

25. $\vec{a}, \vec{b}, \vec{c}$ **will be co-planar if.**
($\vec{a}, \vec{b}, \vec{c}$ एमतलीय होगा यदि–
(a) $(\vec{a} . \vec{b}) \vec{c} = 0$ (b) $\vec{a} . (\vec{b} \times \vec{c}) = 0$ (c) $\vec{a} \times (\vec{b} \times \vec{c}) = 0$ (d) $\vec{a} . (\vec{b} + \vec{c}) = 0$

26. **If A is a 2 × 2 determinant having value 5 then value of $|2A|$ will be**
(यदि A एक 2 × 2 सारणिक है जिसका मान 5 है, तो $|2A|$ का मान होगा ?)
(a) 10 (b) 25 (c) 20 (d) None of these (इनमें से कोई नहीं)

27. $\int \frac{1}{a^2 + x^2} dx$ =
(a) $\sin^{-1} \frac{x}{a}$ (b) $\frac{1}{a} \tan^{-1} \frac{x}{a}$ (c) $\tan^{-1} \frac{x}{a}$ (d) None of these (इनमें से कोई नहीं)

28. **The direction cosine of any normal to the xy plane are**
(xy -तल पर अभिलम्ब का दिक् कोज्या है ?)
(a) (1, 0, 0) (b) (0, 1, 0) (c) (1, 1, 0) (d) (0, 0, 1)

29. **The value of $[\hat{i}\ \hat{j}\ \hat{k}]$ is**
($[\hat{i}\ \hat{j}\ \hat{k}]$ का मान बराबर है ?)
(a) 1 (b) 0 (c) 2 (d) None of these (इनमें से कोई नहीं)

30. **The value of $\int_{-2}^{2} (ax^3 + bx + c) dx$ depends on**
($\int_{-2}^{2} (ax^3 + bx + c) dx$ का मान आश्रित है ?)
(a) a (b) b (c) c (d) None of these (इनमें से कोई नहीं)

31. **If (यदि) $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ then (तो) A^n =**
(a) $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & n \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2^n \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & n \\ 0 & 2 \end{bmatrix}$

32. $|\hat{i}+\hat{j}+\hat{k}|$ =

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 2 (d) None of these (इनमें से कोई नहीं)

33. $P\left(\frac{A}{B}\right)+P\left(\frac{A'}{B}\right)$ =

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) None of these (इनमें से कोई नहीं)

34. If A is a $m \times n$ matrix then order of A' is

(यदि A एक $(m \times n)$ कोटि का आव्युह है तो A' की कोटि होगी ?

(a) $m \times n$ (b) $n \times m$ (c) $n \times n$ (d) None of these (इनमें से कोई नहीं)

35. If (यदि) $y = e^{3\log x}$, then (तो) $\frac{dy}{dx}$=

(a) $3x^2$ (b) $2\log x$ (c) $\frac{3y}{x}$ (d) $3xy$

36. Two lines with d.c. (l_1, m_1, n_1) and (l_2, m_2, n_2) are at right angle if

(दो रेखा जिसकी दिक् कोज्या (l_1, m_1, n_1) और (l_2, m_2, n_2) है, परस्पर लम्बवत होंगे यदि

(a) $l_1l_2 + m_1m_2 + n_1n_2 = 0$ (b) $l_1 = l_2$, $m_1 = m_2$, $n_1 = n_2$

(c) $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$ (d) $l_1l_2 = m_1m_2 = n_1n_2$

37. Differential co-efficient of $\cos\sqrt{x}$ w.r.t x

(x के सापेक्ष में $\cos\sqrt{x}$ का अवकल गुणांक है ?)

(a) $\frac{1}{\sqrt{x}}\sin\sqrt{x}$ (b) $\frac{1}{2\sqrt{x}}\sin\sqrt{x}$ (c) $\frac{-1}{2\sqrt{x}}\sin\sqrt{x}$ (d) $\sin\sqrt{x}$

38. The range of $f(x)=\sqrt{(x-1)(3-x)}$ is

($f(x)=\sqrt{(x-1)(3-x)}$ का परास है ?)

(a) [1, 3] (b) [0, 1] (c) [–2, 2] (d) None of these (इनमें से कोई नहीं)

39. $\frac{d}{dx}\{\cos(\sin x)\}$ = ...

(a) $\sin(\sin x)\cos x$ (b) $-\sin(\sin x)\cos x$ (c) $-\sin(\cos x)\cos x$ (d) None of these (इनमें से कोई नहीं)

40. $\int\frac{\cos\sqrt{x}}{\sqrt{x}}dx$=

(a) $2\cos x + k$ (b) $2\sin x + k$ (c) $2\sin\sqrt{x}+k$ (d) $2\cos\sqrt{x}+k$

41. If (यदि) $P(A) = 0.2$, $P(B/A) = 0.3$, then (तो) $P(A \cap B)$ =

(a) 0.6 (b) 0.006 (c) 0.06 (d) None of these (इनमें से कोई नहीं)

42. $\int\frac{\log x}{x}.dx$=

(a) $\frac{1}{2}(\log x)^2+k$ (b) $-\frac{1}{2}(\log x)^2+k$ (c) $\frac{2}{x^2}+k$ (d) None of these (इनमें से कोई नहीं)

43. Which of the following is not a homogenous differntial Equation.
(निम्न में कौन-सा एक समधातीय अवकल समीकरण नहीं है ?)

(a) $y^2dx + (x^2 + xy)\,dy = 0$ (b) $\frac{dy}{dx}=\frac{y}{x}-\frac{y^3}{x^3}$

(c) $(x - y)\,dy + y^2\,.dx = 0$ (d) $\frac{dy}{dx}=\sin\frac{y}{x}$

44. $\frac{d}{dx}.e^{\frac{1}{x}} =$

(a) $\frac{1}{x}.e^{\frac{1}{x}-1}$ (b) $\frac{-e^{\frac{1}{x}}}{x^2}$ (c) $e^{\frac{1}{x}}.\log x$ (d) None of these (इनमें से कोई नहीं)

45. $\frac{d}{dx}.\log 10^x$

(a) $\frac{1}{x}$ (b) $\frac{1}{x}.\log 10$ (c) $\frac{1}{x\log 10}$ (d) None of these (इनमें से कोई नहीं)

46. If A and B are in dependent event and $P(A) = 0.3$, $P(A\cup B) = .08$ then $P(B)$= .....
(यदि A और B स्वतंत्र घटनाएँ है और $P(A) = 0.3$, $P(A\cup B) = .08$ तो $P(B)$= .....)

(a) $\frac{5}{7}$ (b) $\frac{3}{7}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

47. $\int\frac{1}{\sqrt[3]{x}}dx=$

(a) $\frac{3}{2}x^{2/3}+k$ (b) $\frac{3}{2x^{2/3}}+k$ (c) $\frac{2}{3x^{2/3}}+k$ (d) $\frac{2}{3}.x^{2/3}+k$

48. $\vec{i}\times\vec{k} =$

(a) 0 (b) $\vec{j}$ (c) $-\vec{j}$ (d) 1

49. $\frac{dx}{dx}$

(a) 1 (b) 0 (c) x (d) None of these (इनमें से कोई नहीं)

50. $\frac{d(e^{-x})}{dx} =$

(a) e^{-x} (b) $-e^{-x}$ (c) $\frac{1}{e^{-x}}$ (d) $-\frac{1}{e^{-x}}$

SECTION (खण्ड)–II

Non-Objective Questions (गैर वस्तुनिष्ट प्रश्न)

Time : [2 Hrs **Full Marks : 50**

समय : 2 घंटा **पूर्णांक : 50**

Question number 1 to 22 are of short Answer type. Each question carry 2 marks. Answer any 15 Question.

[प्रश्न संख्या 1 से 22 तक लघुउत्तरीय प्रकार के है । प्रत्येक के लिए 2 अंक निर्धारित है। किन्हीं 15 प्रश्नों के उत्तर दें) $15 \times 2 = 30$

1. **Let R be a relation in the set of integer I defined by aRb iff a & b both are neither even nor odd. Then show that R is symmetric but neither reflexive nor Transitive.**
(माना कि पूर्णांकों के समुच्चय I पर एक सम्बन्ध R है जिसे aRb यदि और केवल यदि a और b दोनों ना तो सम ना ही विषम है, द्वारा परिभाषित है । दिखावें कि R सममित है किन्तु न ही स्वतुल्य ना ही संक्रमक है।

2. **Verify Rolle's theorem on $f(x)=x\sin\frac{1}{x}$, $x\in[-1,1]$**

($f(x)=x\sin\frac{1}{x}$, $x\in[-1,1]$ पर रॉले प्रमेय की सत्यता की जाँच करें ।

3. **Find slope of tangent of the curve $y = ae^{-x/b}$ at the point where it crosses the y-axis.**
(वक्र $y = ae^{-x/b}$ पर स्पर्श रेखा की ढ़ाल उस बिन्दु पर ज्ञात करें जहाँ वक्र y-axis को पार करती है?)

4. **If $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$ then find a unit vector in the direction of $\vec{a}+\vec{b}$**

(यदि $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ और $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$ तो $\vec{a}+\vec{b}$ की दिशा में एक इकाई सदिश ज्ञात करें ।)

5. **If $x^y = e^{x-y}$. Prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$**

(यदि, $x^y = e^{x-y}$ सिद्ध करें $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$)

6. **Six coin are tossed simultanously. Find the probability of getting no head)**
(छः सिक्कों को एक साथ उछाला जाता है तो कोई शीर्ष नहीं आने की प्रायिकता ज्ञात करें।)

7. **Show that (दिखाएँ कि)**

$$\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix} \neq \begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$$

8. **Find the area of the Parallelogram whose adjacent sides are $\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$**

(समानान्तर चतुर्भुज का क्षेत्रफल ज्ञात करें जिसकी परस्पर भुजा $\hat{i}+2\hat{j}+2\hat{k}$ और $2\hat{i}+\hat{j}-\hat{k}$ है।)

9. **A random variable X has following probability function.**
(एक यादृच्छ चर X के निम्न प्रायिकता फलन है ।)

$X=x_i$	0	1	2	3	4	5	6	7
$P(x_i)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Than find (तो ज्ञात करें) P ($x < 2$)

10. If (यदि) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$ then Prove that (तो सिद्ध करें कि) $xy + yz + zx = 1$

11. Integrate (समाकलन करें) : $\int \tan^{-1} x\,dx$

12. Find Integrating factor of differential equation $x\frac{dy}{dx}+y=x^3$

(अवकल समीकरण $x\frac{dy}{dx}+y=x^3$ का समाकलनीय गुणांक ज्ञात करें)

13. Check the continuity of $f(x) = \frac{|x|}{x}, x\neq 0$

$=0, x=0$

($f(x) = \frac{|x|}{x}, x\neq 0$ के सतता की जाँच करें ।)

$=0, x=0$

14. Prove that (सिद्ध करें) : $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$

15. Solve the equation (समीकरण को हल करें)

$$\begin{vmatrix} 15-x & 1 & 10 \\ 11-3x & 1 & 16 \\ 7-x & 1 & 13 \end{vmatrix} = 0$$

16. Show that the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $-4\hat{i}+6\hat{j}-8\hat{k}$ are collinear.

(दिखावें कि सदिश $2\hat{i}-3\hat{j}+4\hat{k}$ और $-4\hat{i}+6\hat{j}-8\hat{k}$ रैखिक है ।

17. Find the angle at which vectors $\hat{i}-\hat{j}+\hat{k}$ are inclined to each of co-ordinate axes.

(सदिश $\hat{i}-\hat{j}+\hat{k}$ का प्रत्येक नियामक अक्ष के साथ झुकाव ज्ञात करें)

18. Find the angle between the plane $-x + y + 2z = 9$ and $x + 2y + z = 5$.
(समतल $-x + y + 2z = 9$ और $x + 2y + z = 5$ के बीच कोण ज्ञात करें।

19. If $P(A) = 0.7$, $P(B) = 0.5$ and $P(B/A) = 0.5$, than find $P(A \cup B)$
(यदि $P(A) = 0.7$, $P(B) = 0.5$ और $P(B/A) = 0.5$ तो $P(A \cup B)$ ज्ञात करें)

20. Write the following as single matrix.
(निम्न को एक आव्युह के रूप में लिखें ।)

$$[1, -2, 3]\begin{bmatrix} 2 & -1 & 5 \\ 0 & 2 & 4 \\ -7 & 5 & 0 \end{bmatrix} - [2, -5, 7]$$

21. If possible find inverse of $f(x)=\frac{x^2}{x^2+1}$

(यदि सम्भव हो, तो $f(x)=\frac{x^2}{x^2+1}$ का व्युत्क्रम ज्ञात करें ।)

22. If $A=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$, show that $(A-A^T)$ is a skew-symmetric matrix.

(यदि $A=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$, तो दिखावें कि $(A-A^T)$ एक असममित आव्युह है ।

प्रश्न संख्या 23 से 26 दीर्घउत्तरीय कोटि के है । प्रत्येक के लिए 5 अंक निर्धारित है । प्रत्येक प्रश्न या उसके विकल्प में से किसी एक का उत्तर दें ।

Question number 23 to 26 are of long answer type. Each question carry 5 marks. Answer every question or alternative of it. [4 × 5]

23. Integrate (समाकलन करे) :

$$\int_0^1 \log\left(\frac{1}{x}-1\right)dx$$

Or, (या)

Using Integration find area enclosed by the circle of radius a.

(समाकलन का उपयोग कर a त्रिज्या वाले वृत्त से घिरे क्षेत्र का क्षेत्रफल ज्ञात करें ।)

24. Solve the differential equation

$$\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$$

Or, (या)

Show that the maximum value of $x+\frac{1}{x}$ is less than its minimum value.)

(दिखावें कि $x+\frac{1}{x}$ का अधिकतम मान, इसके न्यूनतम मान से कम हैं।)

25. Find the equation of the plane which is at a distance of 5 units from the origin & is perpendicular to $2\hat{i}-3\hat{j}+6\hat{k}$

($2\hat{i}-3\hat{j}+6\hat{k}$ पर लम्बवत और मूलबिन्दु से 5 इकाई की दूरी पर समतल का समीकरण ज्ञात करें ।

Or, (या)

The cartesian equation of a line is $3x+1=6y-2=1-z$. Find its direction ratio and equation in vector form.

(किसी रेखा का कार्त्तीय समीकरण $3x+1=6y-2=1-z$ है, तो उसका दिक् अनुपात और सदिश रूप

में समीकरण ज्ञात करें ।)

26. **Solve the LPP graphically**

Minimize (न्यूनतमीकरण) $z = x - 5y + 20$

Subjected to constraints (जबकि)

$$x - y \geq 0$$
$$-x + 2y \geq 2$$
$$x \geq 3$$
$$y \leq 4$$
$$x, y \geq 0$$

Or, (या)

A furniture dealer deals in only two items—tables & chairs. He has Rs. 1000 to invest and a space to store atmost 60 pieces. A table costs him Rs. 500 and a chair costs him Rs. 100. He can sell a table at *Rs*. 550. and a chair at Rs. 115.. Assume that he can sell all the items that he buys. Formulate this problem as a LPP so that he maximizes his profit.

(एक बढ़ई केवल दो तरह के आटम का व्यावसय करता है। उसके पास 1000 रुपये व्यावसाय में लगाने और अधिक से अधिक 60 आइटम को स्टोर करने की जगह है। एक टेबुल की कीमत 500 रुपये और कुर्सी की कीमत 100 रुपये ह। वह टेबुल को 550 रुपये और कुर्सी को 115 रुपये में बेचता है। यह मानते हुए कि वह खरीदे गये सभी समान को बेचता है, इस समस्या का सुत्रीकरण अधिकतम लाभ हेतु करें ।)

SOLUTION
Answer Key

Objective Question :

1. (a)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (b)
11. (a)	**12.** (b)	**13.** (a)	**14.** (c)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (a)	**20.** (b)
21. (b)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (c)	**27.** (b)	**28.** (d)	**29.** (a)	**30.** (c)
31. (a)	**32.** (b)	**33.** (a)	**34.** (b)	**35.** (a)	**36.** (a)	**37.** (c)	**38.** (a)	**39.** (b)	**40.** (b)
41. (c)	**42.** (a)	**43.** (c)	**44.** (b)	**45.** (c)	**46.** (a)	**47.** (a)	**48.** (d)	**49.** (a)	**50.** (b)

Solution to Short Answer type question :

1. Given relation aRb is defined only when, both a & b are not even or odd at a time.
i.e., if a is even & b is odd then aRb is well defined.

(i) Let a and b be odd and even then aRb is defined and $aRb \Rightarrow bRa$
Hence, R is symmetric

(ii) Let a be an odd then aRa is not defined and a be an even then also aRa is not defined
So, R is not Reflexive

(iii) Let a & b be odd and even respectively. Then if R is transitive, then $aRb, bRa \Rightarrow aRa$ but, then aRa is not defined as a is odd. Hence, R is not transitive.

2. Given function is

$f(x) = x \sin\frac{1}{x}, x \in [-1, 1]$

$f(x) = 0$

Now $\lim_{x\to 0} f(x) = \lim_{x\to 0} x\sin\frac{1}{x} = 0 \times$ a number between 1 and $-1 = 0 = f(0)$

$\therefore$ $f(x)$ is continuous at $x = 0$

Again, left hand derivative $Lf'(0) = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h}$

$$= \lim_{h\to 0} \frac{-h\sin\left(-\frac{1}{h}\right)-0}{-h} = \lim_{h\to 0}\left(-\sin\frac{1}{h}\right)$$

Which deosnot exist

$\therefore$ The given function is not derivable at $x = 0$

So, Rolle's Theorem is not applicable.

3. The curve cuts y-axis at $x = 0$
When $x = 0, y = ae^0 = a$

Now, $\frac{dy}{dx} = ae^{-x/b}.\left(-\frac{1}{b}\right)$

$\therefore$ $\left(\frac{dy}{dx}\right)_{(0,a)} = ae^0\left(-\frac{1}{b}\right) = -\frac{a}{b}$

Hence, required slope of tangent of curve is $\left(-\frac{a}{b}\right)$

4. Here $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$

and $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$

$\therefore \quad \vec{a}+\vec{b} = \hat{i}+0.\hat{j}+\hat{k}=\hat{i}+\hat{k}$

$\therefore \quad |\vec{a}+\vec{b}|=\sqrt{1^2+0^2+1^2}=\sqrt{2}$

So, unit vector in the direction of $\vec{a}+\vec{b}=\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}=\frac{\hat{i}+\hat{k}}{\sqrt{2}}=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{2}\hat{j}$

5. Given, $x^y = e^{x-y}$

Taking Logarithm, we get

$y \log x = (x-y)\log e^e$

$\Rightarrow y \log x = x - y$

$\Rightarrow y(\log x + 1) = x \Rightarrow y = \frac{x}{1+\log x}$

Differentiating w.r.t 'x', we have

$$\frac{dy}{dx} = \frac{(1+\log x).1-x\left(0+\frac{1}{x}\right)}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$$

6. Let P be the Probability of getting a head in the toss of a coin.

Then $P = \frac{1}{2}$

Let q = prob of not getting a head = 1 – P = $\frac{1}{2}$

Let X = Number of success in the experiment

So, X can take value 0, 1, 2, 3, 4, 5, 6 *i.e.*, $n = 6$

Now $P(x = r) = {}^n c_r\, p^r.q^{n-r}$

So, P(no head) = P($x = 0$) = ${}^6c_0\left(\frac{1}{2}\right)^0.\left(\frac{1}{2}\right)^{6-0} = \frac{1}{64}$

7. $\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix} = \begin{bmatrix}10-3 & 5-4\\ 12+21 & 6+28\end{bmatrix} = \begin{bmatrix}7 & 1\\ 33 & 34\end{bmatrix}$ ...(i)

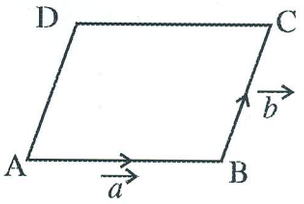
and $\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix} = \begin{bmatrix}10+6 & -2+7\\ 15+24 & -3+28\end{bmatrix} = \begin{bmatrix}16 & 5\\ 39 & 25\end{bmatrix}$ ...(ii)

From(i) and (ii), we get

$$\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix} \neq \begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$$

8. Let ABCD be the parallelogram whose sides are

$\vec{AB} = \vec{a} = \vec{i} + 2\vec{j} + 2\vec{k}$ and $\vec{BC} = \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$



Now, $\vec{a} \times \vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\1 & 2 & 2\\2 & 1 & -1\end{vmatrix}$

$= \hat{i}(-2-2) - \hat{j}(-1-4) + \hat{k}(1-4)$

$= -4\hat{i} + 5\hat{j} - 3\hat{k}$

So area of parallelogram ABCD $= \left|\vec{a} \times \vec{b}\right| = \left|-4\hat{i} + 5\hat{j} - 3\hat{k}\right| = \sqrt{50} = 5\sqrt{2}$ sq. unit

9. Since, sum of all the probabilities of a probability distribution is 1

So, $P(x=0) + P(x=1) + P(x=2) + \ldots\ldots\ldots + P(x=7) = 1$

$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow 10k^2 + 9k = 0$

$\Rightarrow (10k-1)(k+1) = 0$

$\therefore \quad k = \frac{1}{10}, -1$

but, $k > 0 \quad \therefore k = \frac{1}{10}$

Hence, $P(x<2) = P(x=0) + P(x=1)$

$= 0 + k = k = \frac{1}{10}$

10. Given that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}x + \tan^{-1}y = \frac{\pi}{2} - \tan^{-1}z$

$\Rightarrow \tan^{-1}\frac{x+y}{1-xy} = \cot^{-1}z = \tan^{-1}\frac{1}{z}$

$\Rightarrow \frac{x+y}{1-xy} = \frac{1}{z}$

$\Rightarrow xz + yz = 1 - xy$

$\Rightarrow xy + yz + zx = 1$

11. $\int \tan^{-1}x\,dx = \int 1.\tan^{-1}x\,dx = \tan^{-1}x\int 1.dx - \int \frac{1}{1+x^2}.x.dx$

$$= x\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2}dx$$

$$= x\tan^{-1}x - \frac{1}{2}\log(1+x^2) + k$$

12. Given differential equation may be written as

$$\frac{dy}{dx} + \frac{1}{x}.y = x^2$$

This is linear differential equation of form $\frac{dy}{dx} + Px = Q$

Here, $P = \frac{1}{x}$, $Q = x^2$

$$\text{I.F.} = e^{\int P dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

13. Given function can be written as

$$f(x) = \begin{cases} \frac{x}{x} = 1, & x > 0 \\ \frac{-x}{x} = -1, & x < 0 \\ 0, & x = 0 \end{cases}$$

Here, $f(0) = 0$

Now, LHL $= \lim_{x\to 0^-} f(x) = \lim_{x\to 0}(-1) = -1$

So, $\lim_{x\to 0^-} f(x) \neq f(0)$

hence, given function is not continuous at $x = 0$

14. LHS $= \tan^{-1}\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{\frac{2}{3}}{\frac{8}{9}} + \tan^{-1}\frac{1}{7}$

$$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}.\frac{1}{7}} = \tan^{-1}1 = \frac{\pi}{4} = \text{RHS}$$

15. Fiven that $\begin{vmatrix} 15-x & 1 & 10 \\ 11-3x & 1 & 16 \\ 7-x & 1 & 13 \end{vmatrix} = 0$

or, $\begin{vmatrix} 15-x & 1 & 10 \\ -4-2x & 0 & 6 \\ -8 & 0 & 3 \end{vmatrix} = 0$

or, $(-1)(-12-6x+48) = 0$

or, $6x - 36 = 0 \therefore x = 6$

16. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$

Hence, $\vec{a}$ and $\vec{b}$ are collinear.

17. Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, then

$$\left|\vec{a}\right| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

The unit vector in the direction of $\vec{a}$ is given by

$$\hat{a} = \frac{\vec{a}}{\left|\vec{a}\right|} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

$\therefore\ l = \cos\alpha = \frac{1}{\sqrt{3}}, m = \cos\beta = -\frac{1}{\sqrt{3}}, n = \cos\gamma = \frac{1}{\sqrt{3}}$

$\Rightarrow \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right), \beta = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right), \gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

18. Given planes are:

$-x + y + 2z = 9$ .....(i)

and $x + 2y + z = 5$ .....(ii)

The direction cosine of the normal to the plane (1) are $\left(\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$ and that of plane (2) are $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$. Let θ be the angle between given plane, then

$$\cos\theta = \left(-\frac{1}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{6}}\right) + \left(\frac{1}{\sqrt{6}}\right)\left(\frac{2}{\sqrt{6}}\right) + \left(\frac{2}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{6}}\right)$$

$$= -\frac{1}{6} + \frac{2}{6} + \frac{2}{6} = \frac{1}{2}$$

$\therefore\ \theta = 60^\circ$

19. $\because\ P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \cap B)}{0.7}$

$\Rightarrow \quad 0.5 = \dfrac{P(A \cap B)}{0.7} \quad \therefore P(A \cap B) = 0.5 \times 0.7 = 0.35$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.5 - 0.35$

$= 1.2 - 0.35 = 0.85$

20. $[1,-2,3].\begin{bmatrix} 2 & -1 & 5 \\ 0 & 2 & 4 \\ -7 & 5 & 0 \end{bmatrix} = [-19, 10, -3]$

$\therefore \quad [1,-2,3].\begin{bmatrix} 2 & -1 & 5 \\ 0 & 2 & 4 \\ -7 & 5 & 0 \end{bmatrix} - [2,-5,7] = [-19, 10, -3] - [2, -5, 7]$

$= [-21, 15, -10]$

21. For Existence of Inverse, function must be one-one and onto

Clearly, $f(-1) = f(I) = \dfrac{1}{2}$

Hence, f is *a* many one function

Therefore f^{-1} does not exist.

22. $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix}$

Let $B = A - A^T = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Clearly, B is a Skew – Symmetric matrix

Hence, $A - A^T$ is Skew – Symmetrix

Long Answer Type Question (दीर्घ उत्तरीय प्रश्न)

23. Let $f(x) = \log\left(\dfrac{1}{x} - 1\right) = \log\left(\dfrac{1-x}{x}\right)$ .....(i)

Then, $f(1-x) = \log\left(\dfrac{1-(1-x)}{1-x}\right) = \log\dfrac{x}{1-x}$ .....(ii)

By, (i) + (ii)

$$f(x) + f(1-x) = \log\left(\frac{1-x}{x}\right) + \log\left(\frac{x}{1-x}\right) = \log 1 = 0$$

$$\therefore \quad 2I = \int_0^1 (f(x) + f(1-x))dx = \int_0^1 0.dx = 0$$

$\therefore \quad I = 0$

Or,

Equation of circle with radius '*a*' is

$x^2 + y^2 = a^2$ .....(i)

Area of circle is 4 times the area of AOBA.

$\therefore$ Area of circle = 4 × area of AOBA

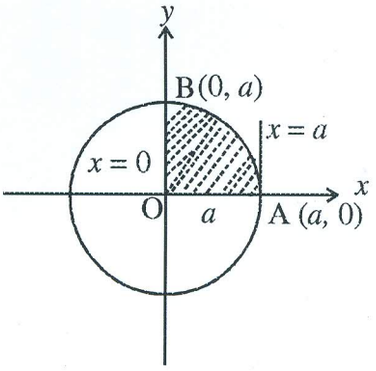
$$= 4\int_0^a x.dy = 4\int_0^a \sqrt{a^2 - y^2}\,dy$$

$$[\because x^2 + y^2 = a^2 \therefore x = \sqrt{a^2 - y^2}\,]$$

$$= 4\left[\frac{y\sqrt{a^2 - y^2}}{2} + \frac{a^2}{2}.\sin^{-1}\frac{y}{a}\right]_0^a$$

$$= 4\left[0 + \frac{a^2}{2}.\sin^{-1}(1) - (0+0)\right]$$

$$= 4.\frac{a^2}{2}.\frac{\pi}{2} = \pi a^2 \text{ sq. unit}$$



24. Given differential equation is $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

or, $\frac{1}{1+y^2}dy = \frac{1}{1+x^2}dx$

on Integration,

$$\int\frac{1}{1+y^2}dy = \int\frac{1}{1+x^2}dy$$

or, $\tan^{-1}y = \tan^{-1}x + C$

or, $\tan^{-1}y - \tan^{-1}x = C$

or, $\tan^{-1}\frac{y-x}{1+yx} = C$

or, $\frac{y-x}{1+yx} = \tan C$

$\therefore$ $\frac{y-x}{1+yx} = k$ (constant) is the require solution

OR

Let $y = x + \frac{1}{x}$

$\therefore$ $\frac{dy}{dx} = 1 - \frac{1}{x^2}$

or, $\frac{dy}{dx} = \frac{x^2 - 1}{x^2}$

For maximum or minimum value of y, $\frac{dy}{dx} = 0$

$\therefore \quad \frac{1-x^2}{x^2} = 0$

$\Rightarrow 1 - x^2 = 0$

$\Rightarrow x = 1, -1$

Again, $\frac{d^2y}{dx^2} = \frac{2}{x^3}$

At, $x = 1$, $\frac{d^2y}{dx^2} = 2 > 0$

So, y has minimum value at $x = 1$

and minumum value of $y = x + \frac{1}{x} = 1 + \frac{1}{1} = 2$

and At, $x = -1$, $\frac{d^2y}{dx^2} = \frac{2}{(-1)^3} = -2 < 0$

So, y has maximum value at $x = -1$

and maximum value of $y = -1 + \frac{1}{-1} = -2$

So, Minimum value of given function is greater than maximum value.

25. Let $\hat{n}$ be the unit vector in the direction of $2\hat{i} - 3\hat{j} + 6\hat{k}$

$\therefore \quad \hat{n} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{2^2 + (-3)^2 + 6^2}} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7} = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be the position vector of any point (x, y, z) on the plane. Then the equation of the plane is

$\hat{n} \cdot \vec{r} = P$

or, $\left(x\hat{i} + y\hat{j} + z\hat{k}\right) \cdot \left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = 5$

or, $2x - 3y + 6z = 35$

OR

Given line is $3x + 1 = 6y - z = 1 - z$ .....(i)

This can be written as

$$\frac{x+\frac{1}{3}}{\frac{1}{3}} = \frac{y-\frac{1}{3}}{\frac{1}{6}} = \frac{z-1}{-1}$$

or, $$\frac{x-\left(-\frac{1}{3}\right)}{\frac{1}{3}} = \frac{y-\frac{1}{3}}{\frac{1}{6}} = \frac{z-1}{-1} \quad \text{.....(ii)}$$

The direction ratio of line are $\left(\frac{1}{3}, \frac{1}{6}, -1\right)$

Also, it passes through the point $\left(-\frac{1}{3}, \frac{1}{3}, 1\right)$

So, Equation of line in vector form is

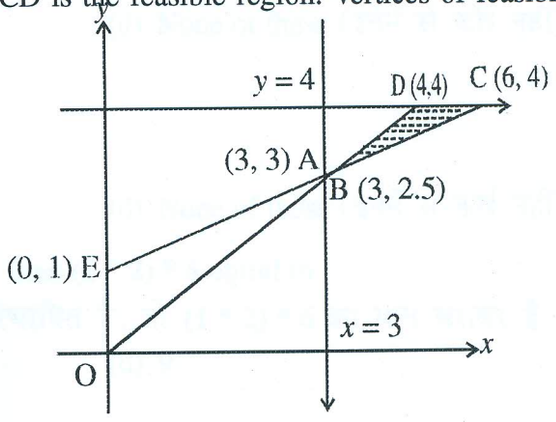
$$\vec{r} = -\frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k} + t\left(\frac{1}{3}\hat{i} + \frac{1}{6}\hat{j} - \hat{k}\right)$$

or, $$\vec{r} = -\frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k} + \lambda\left(2\hat{i} + \hat{j} - 6\hat{k}\right) \text{ where } \lambda = \frac{t}{6}$$

26. We first draw the lines

$x - y = 0$
$-x + 2y = 2$
$x = 3$
$y = 4$

Here, the shaded region is the feasible region i.e. ABCD is the feasible region. Vertices of feasible region are A (3, 3), B (3, 2.5), C (6, 4) and D (4, 4)



Given that $z = x - 5y + 20$

At A(3, 3), $z = 8$, At B(3, 2.5), $z = 10.5$

At C(6, 4) $z = 6$ At D(4, 4), $z = 4$

The minimum value of z is at pt. D(4, 4)

So, Minimum value of $z = 4$

OR,

Let x be the number of tables and y be the number of chairs that the dealer buys and sells.

The profit on table = Rs. (550 – 500) = Rs. 50

and the profit on a chair = Rs. (115 – 100) = Rs. 15

So, Total profit $z = 50x + 15y$

Thus, the mathematical form of given LPP is

maximize $z = 50x + 15y$

Subjected to, $x + y \leq 60$

$5x + y \leq 100$

and $x \geq 0, y \geq 0$.